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# EXPLORING DEGENERATE BAND EDGE MODE IN HPM TRAVELING TUBE

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# MAIN OBJECTIVES FOR THE FIRST YEAR

- Explore degenerate band edge (DBE) modes for multidimensional transmission lines and waveguides.

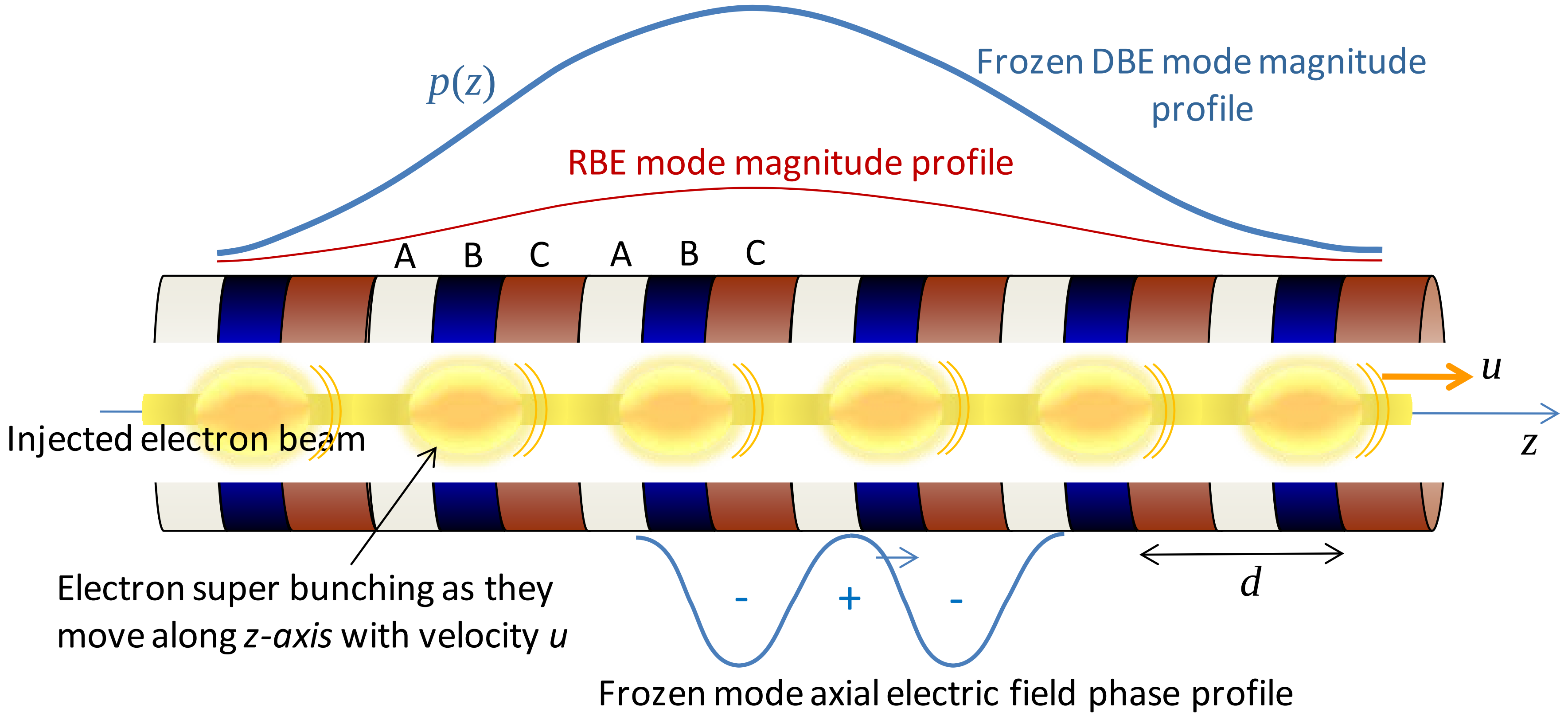
- DBE mode with alternating axial electric field .

- Transmission line model of TWT that can account for significant feature of the amplification.

- Suggested design of realistic waveguide for HPM TWT supporting DBE.

# TWT with super amplification via DBE Mode

TWT with super amplification via the DBE mode.
A, B, and C are three different waveguide sections with distinct transverse anisotropy.



# FROZEN MODE REGIMES

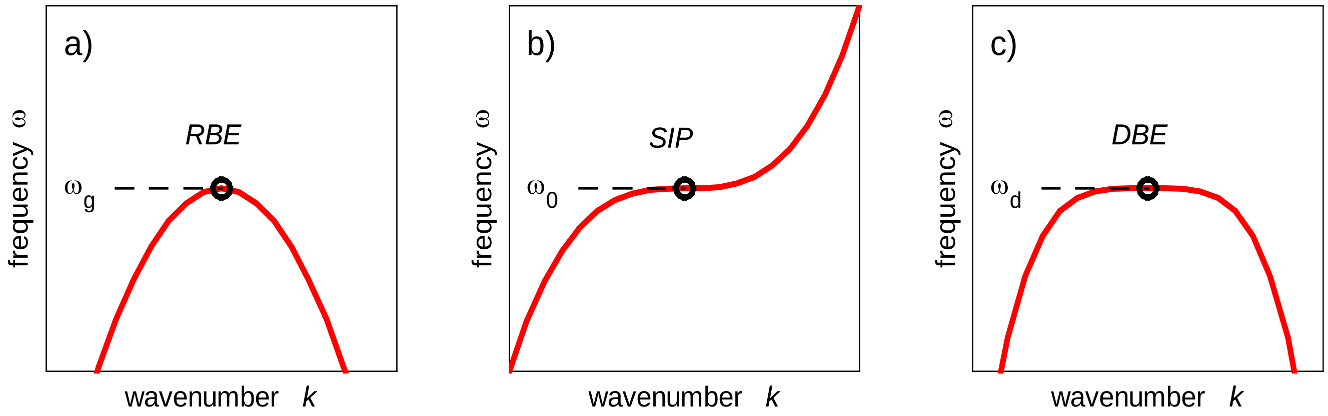
# Stationary points of the dispersion relation. Slow waves.

$$v_g = \frac{\partial \omega}{\partial k} = 0, \quad \text{at } \omega = \omega_s = \omega(k_s).$$

1. Dramatic increase in density of modes.
2. Qualitative changes in the eigenmode structure (can lead to the frozen mode regime).

## Examples of stationary points:

- Regular band edge (RBE): $\omega - \omega_g \propto (k - k_g)^2, \quad v_g \propto (k - k_g) \propto (\omega - \omega_g)^{1/2}.$
- Stationary inflection point (SIP): $\omega - \omega_0 \propto (k - k_0)^3, \quad v_g \propto (k - k_0)^2 \propto (\omega - \omega_0)^{2/3}.$
- Degenerate band edge (DBE): $\omega - \omega_d \propto (k - k_d)^4, \quad v_g \propto (k - k_d)^3 \propto (\omega - \omega_d)^{3/4}.$




Each stationary point is associated with slow wave, but there are some fundamental differences between these three cases.

# BASIC CHARACTERISTIC OF THE FROZEN MODE REGIME

- The frozen mode regime is not a conventional resonance – it is not particularly sensitive to the shape and dimensions of the structure.

- The frozen mode regime is much more robust than a common resonance.

- The frozen mode regime persists even for relatively short pulses (bandwidth advantage).

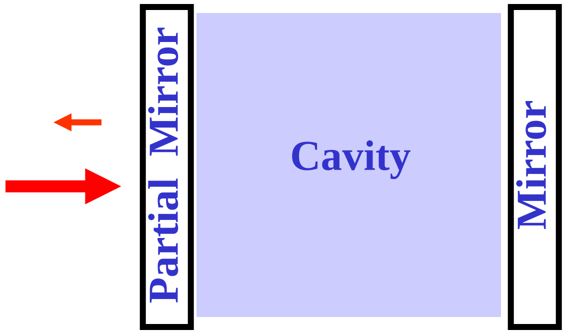
# SLOW WAVE RESONANCE

Slow-wave phenomena in bounded photonic crystals.

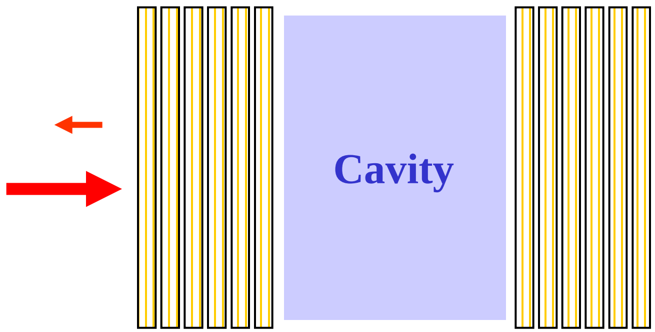
# Cavity Resonator *vs.* Slow Wave Resonator

## Examples of Plane-Parallel Open Resonators

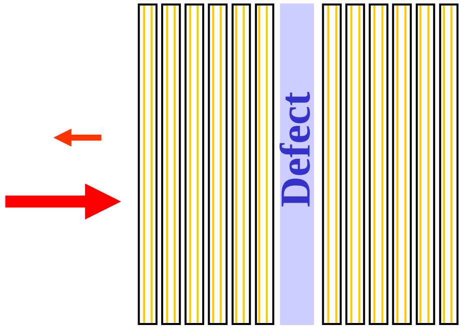
Simplest uniform resonance cavity with metallic reflectors



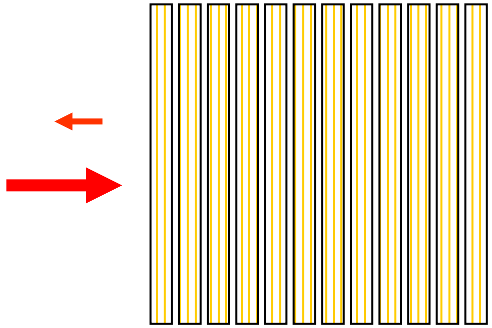

Uniform resonance cavity with photonic reflectors (DBR)




Single mode photonic cavity



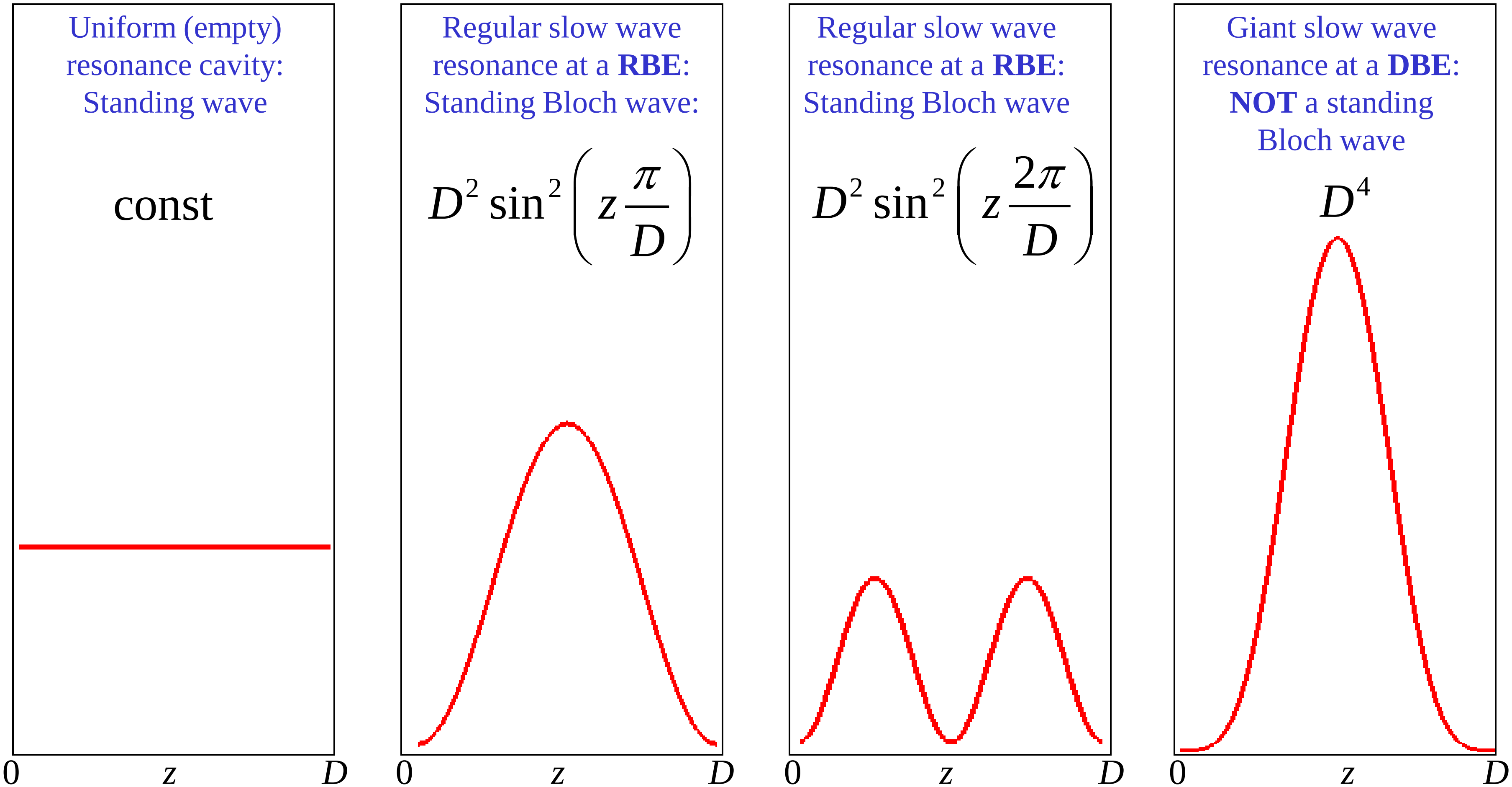

**Slow wave photonic resonator**
(no reflectors needed)



# EM energy density distribution at resonance frequency



$$W(z) = \frac{1}{8\pi}\left[\varepsilon E^2(z) + \mu H^2(z)\right]$$



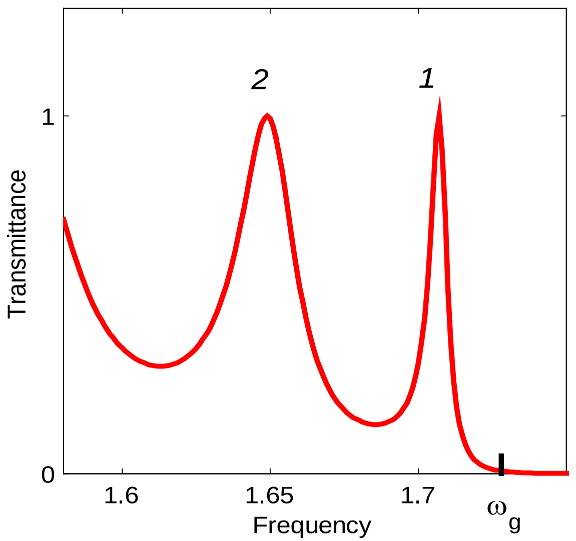

Poor confinement

Better confinement

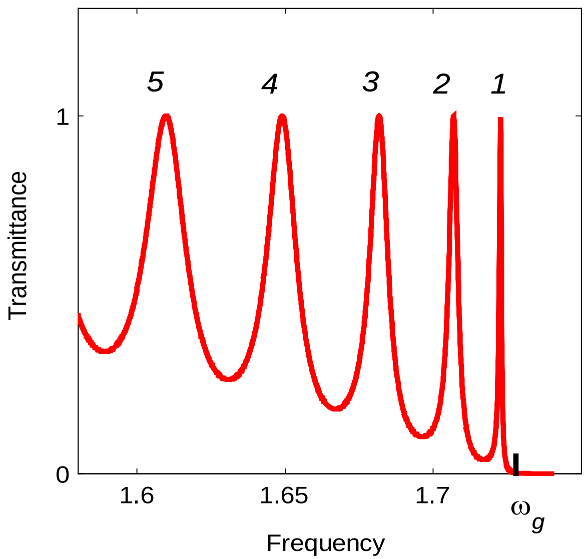
Best confinement

# Transmission band edge resonances near a RBE

a) *N = 16*



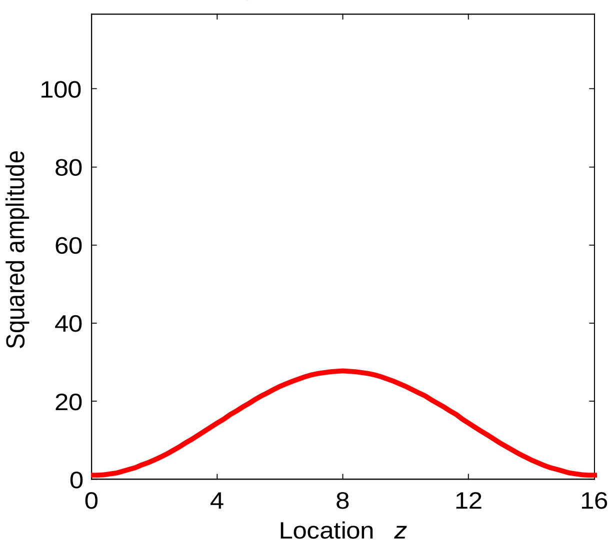

b) *N = 32*



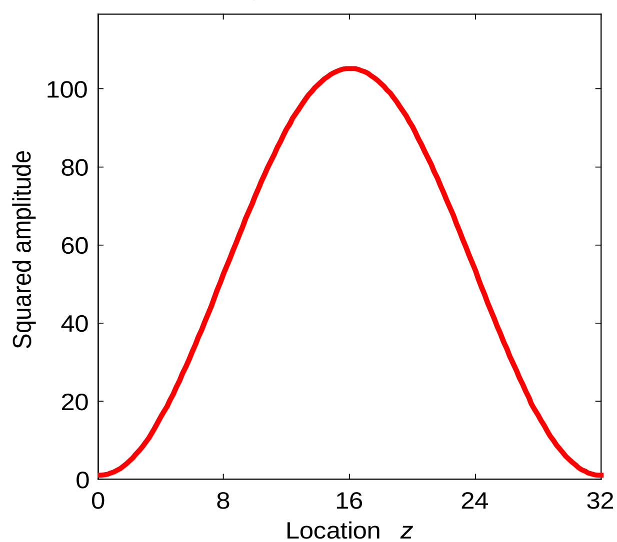

Transmission dispersion of periodic stacks with different $N$.

$\omega_g$ – the RBE frequency

a) *N = 16, s = 1*




b) *N = 32, s = 1*

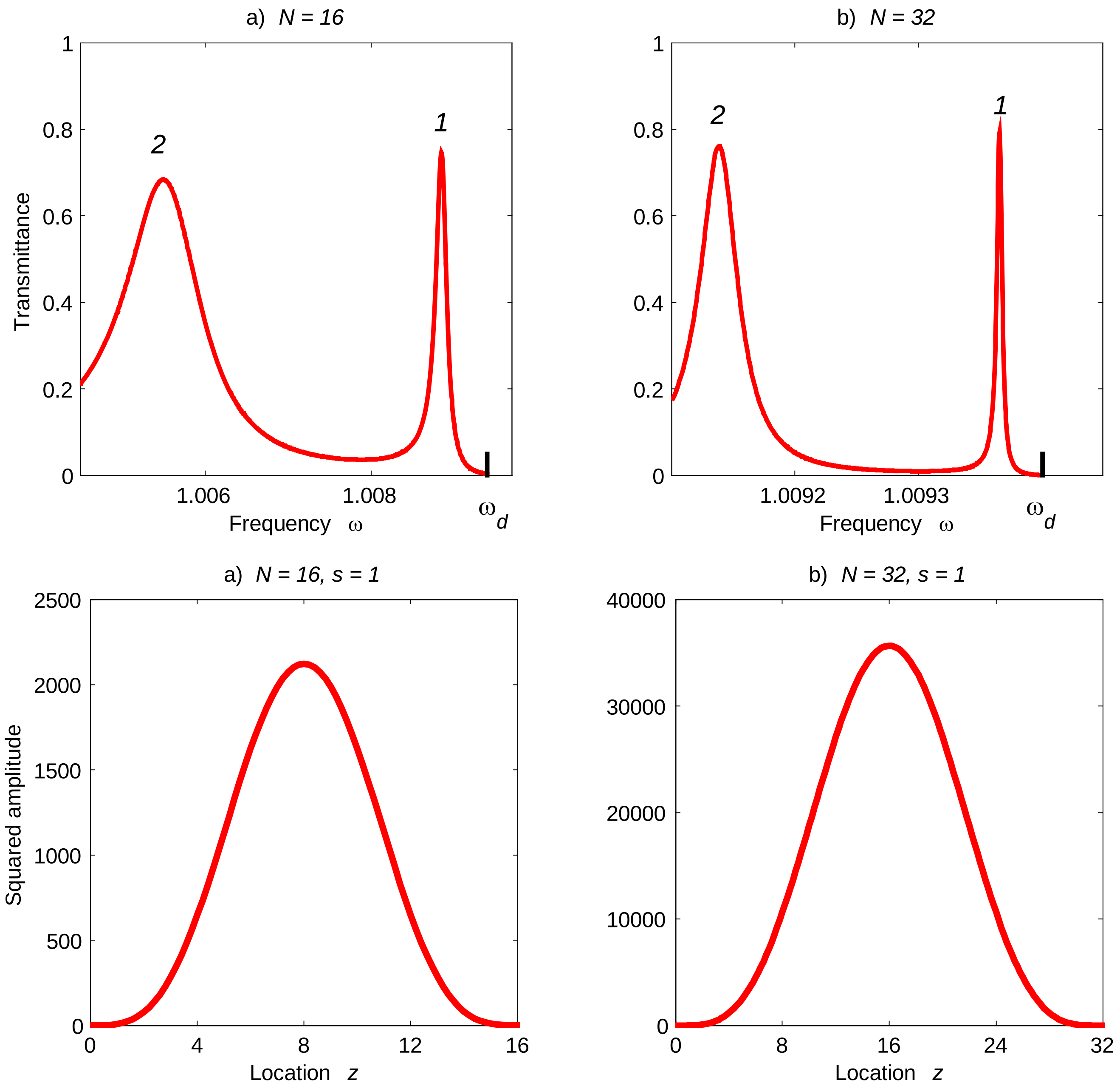



Smoothed energy density distribution at frequency of the first resonance

$$\max\left(W\right) \propto W_I N^2$$

# Giant transmission band edge resonances near a DBE






Transmission dispersion of periodic stacks with different $N$.

$\omega_d$ – the DBE frequency

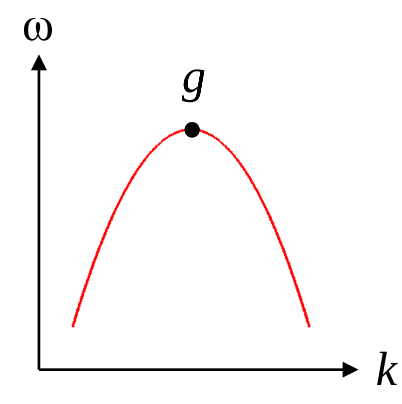
Smoothed Field intensity distribution at frequency of first transmission resonance

$$\max(W) \propto W_I N^4$$

# **Summary:** RBE resonator vs. DBE resonator

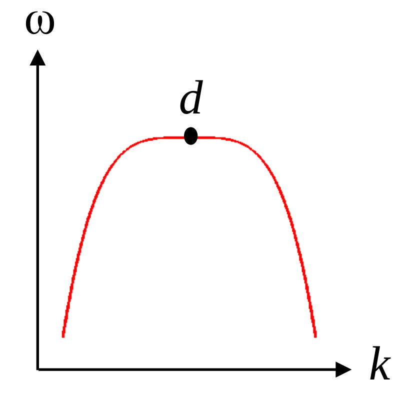
Regular Band Edge: $\omega \approx \omega_g - \frac{a_2}{2}\left(k - k_g\right)^2$ :

$$\max(W) \propto W_I \left(\frac{N}{m}\right)^2$$

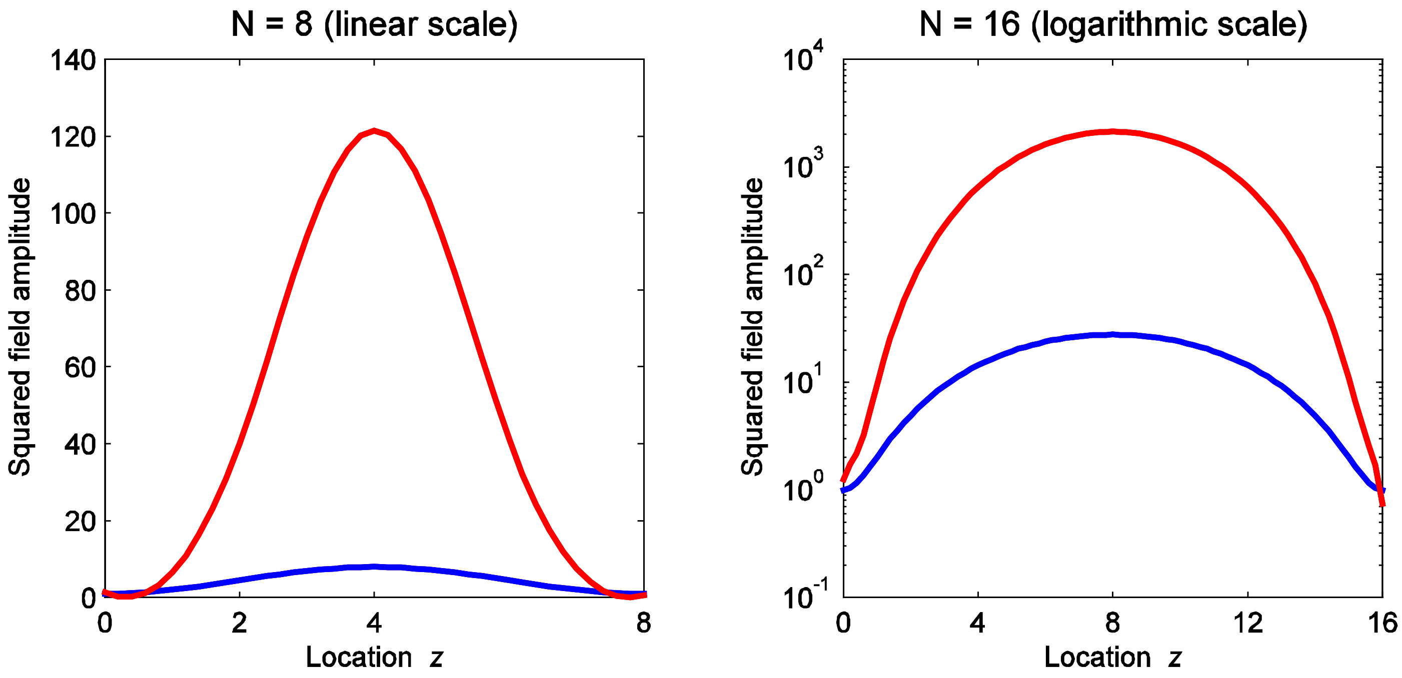



Degenerate Band Edge: $\omega \approx \omega_d - \frac{a_4}{4}\left(k - k_d\right)^4$ :

$$\max(W) \propto W_I \left(\frac{N}{m}\right)^4$$



# *Example*: Slow-wave cavity resonance in periodic stacks composed of different number $N$ of unit cells.




Energy density distribution inside photonic crystal at frequency of slow wave resonance

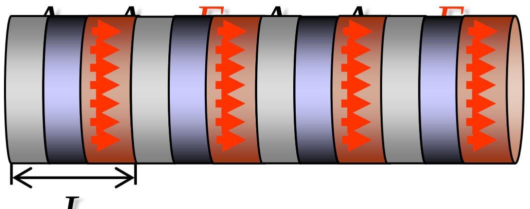
Regular Band Edge: $\max(W) \propto W_I N^2$

Degenerate Band Edge: $\max(W) \propto W_I N^4$

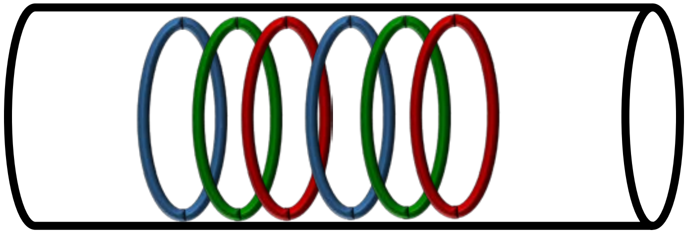
A DBE slow-wave resonator composed of $N$ layers performs similar to a standard RBE resonator composed of $N^2$ layers, which implies a huge size reduction.

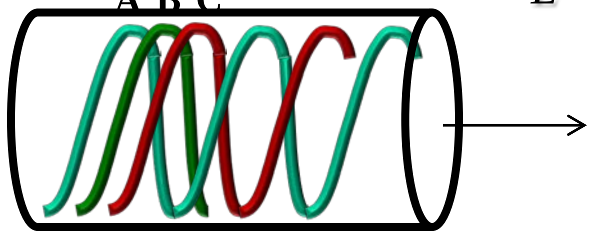
# Floquet expansion of fields

- The electric field in periodic structures (periodic except for an inter-element phase shift):




$$\mathbf{E}(\mathbf{r}+d\hat{\mathbf{z}},k_z)=\mathbf{E}(\mathbf{r},k_z)e^{ik_z d}$$



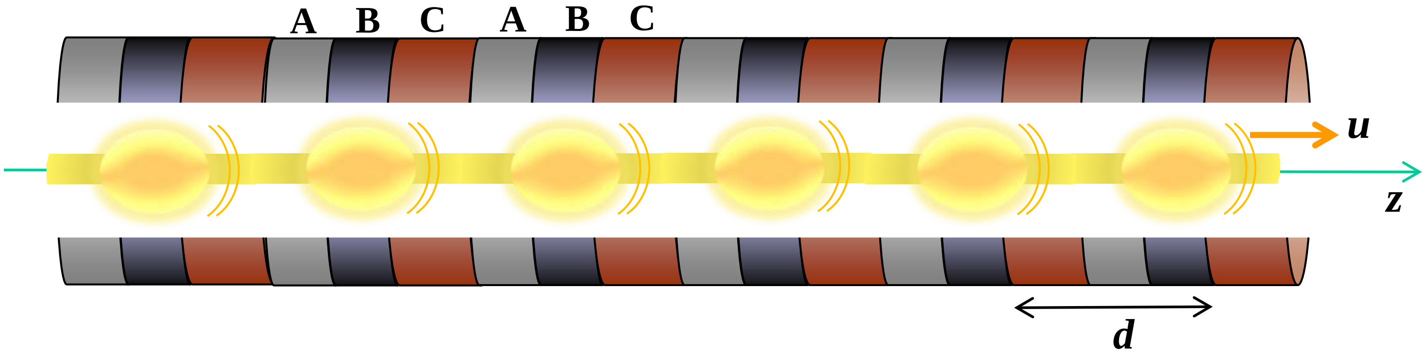

- A mode is expressed in term of Fourier series expansion, and thus represented as the superposition of Floquet spatial harmonics

$$\mathbf{E}^{\text{mode}}(\mathbf{r},k_z)=\sum_{p=-\infty}^{\infty} e^{ik_{z,p}z}\mathbf{e}_p^{\text{mode}}(x,y,k_z)$$

$$k_{z,p}=k_z+2\pi p/d$$

$$k_{z,p}=\beta_{z,p}+i\alpha_z$$

# Physical modes for coupling




- Forward/Backward

$$k_{z,p} = \beta_{z,p} + i\alpha_z \Rightarrow \begin{cases} \beta_{z,p}\alpha_z > 0 & \text{Forward waves} \\ \beta_{z,p}\alpha_z < 0 & \text{Backward waves} \end{cases}$$

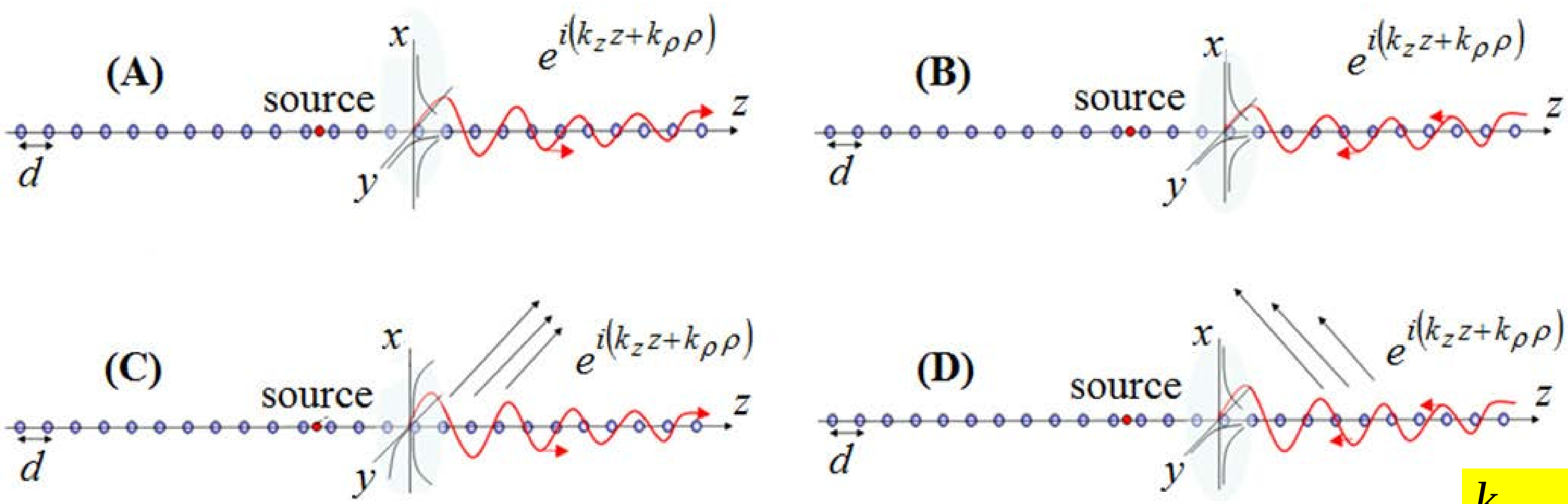
- Slow/Fast (coupling with field produced by electron bunches)

Slow Mode: *all* its Floquet wavenumbers are outside the "visible" region, or

$$\left|\beta_{z,p}\right| > k$$

Fast Mode: mode has *at least one* Floquet wavenumber within the "visible" region, or $\left|\beta_{z,p}\right| < k$

# Physical waves in open periodic structures




$$k_{z,p} = \beta_{z,p} + i\alpha_z$$

$$\beta_{z,p} = k_z + \frac{2\pi p}{d}$$

| | **Forward Wave** $\beta_{z,p}\alpha_z > 0$ | **Backward Wave** $\beta_{z,p}\alpha_z < 0$ |
|---|---|---|
| **Slow Wave** | **(A)** $\lvert\beta_{z,p}\rvert > k$, $\alpha_{\rho,p} > 0$ (proper, bound) | **(B)** $\lvert\beta_{z,p}\rvert > k$, $\alpha_{\rho,p} > 0$ (proper, bound) |
| **Fast Wave** | **(C)** $\lvert\beta_{z,p}\rvert < k$, $\alpha_{\rho,p} < 0$ (improper, leaky) | **(D)** $\lvert\beta_{z,p}\rvert < k$, $\alpha_{\rho,p} > 0$ (proper, leaky) |

Theory is complicated, but it can be summarized

# Methods for complex mode calculations

Peculiar modes investigated here need some fine determination:

- complex wavenumber or complex frequency descriptions
- pairing of modes (long discussion in literature)
- spectral points with vanishing derivative
- time domain description of polarization

## Methods:

- Green's function methods, combined with method of moments (MoM)
- Mode matching (field expansions)
- Commercial software is not able to determine complex modes, but it can be combined with properties of complex modes (i.e., moving around constraints of commercial software, HFSS, CST, FEKO, NEC)
- Analytic and physical properties

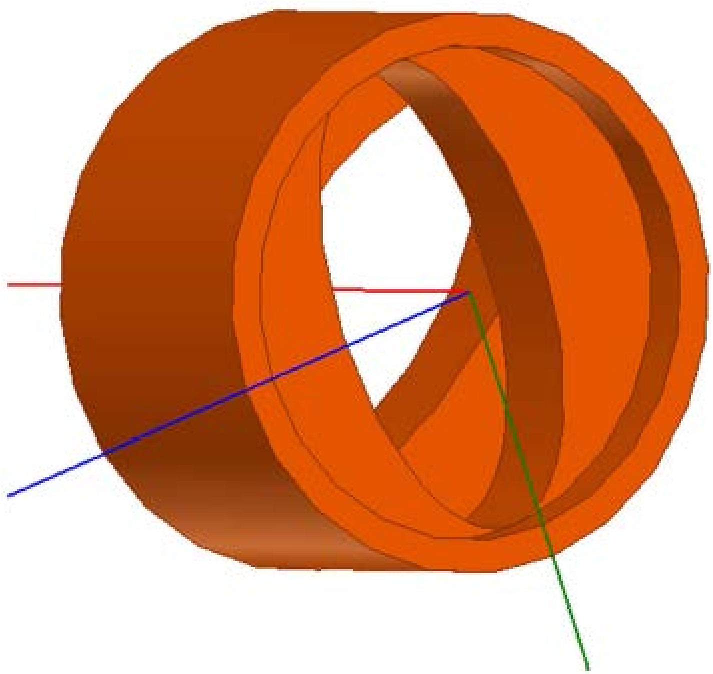
# Points to be developed

- Field in periodic structures
- Complex modes in periodic structures
- Peculiar spectral points (RBE, SIP, DBE)
- Possible structures exhibiting peculiar points
- Excitation of complex modes in periodic structures and in truncated periodic structures
- Coupling of modes with fields produced by electron bunches
- Understanding complex modes in the time domain, including polarization evolution

# Modes



- Waveguide with elliptical sections



The elliptical cross sections may act as anisotropic sections

| Mode | Ratio of Second Derivative Zero Crossing | Ratio of Third Derivative Zero Crossing |
|---|---|---|
| 1 | None | None |
| 2 | 2.85 | 2.90 |
| 3 | 2.90 | 3.00 |
| 4 | 2.10 | 2.60 and 2.80 |
| 5 | 2.30 | 2.45 |

# Analyzing Modes

Vanishing derivatives (up to the third one)

| Mode | Ratio of Second Derivative Zero Crossing | Ratio of Third Derivative Zero Crossing |
|---|---|---|
| 1 | None | None |
| 2 | 2.85 | 2.90 |
| 3 | 2.90 | 3.00 |
| 4 | 2.10 | 2.60 and 2.80 |
| 5 | 2.30 | 2.45 |