Time Domain Response of Split-Ring Resonators in Waveguide Below Cut-Off Structure

M. Aziz Hmaidi, Mark Gilmore

MURI Teleconference
01/06/2017

University of New Mexico, Electrical and Computer Engineering Department
Presentation Outline

• Introduction

• Summary of previous results

• Modeling of the SRR filled WG system as an LTI system

• Similarities between previous results and model

• Model comparison with Genetic algorithm

• Model limits

• Conclusion and Ongoing/Future work
Introduction

• Interest in MTM has been increasing since the 1st model by J. Pendry

• We are interested in this MURI in developing and understanding several MTM structures in high power in order to take advantage of the MTM properties and the compactness of the structures

• For short pulse devices, understanding the MTM time-behavior is crucial. For this purpose, UNM EM-group is studying, designing and characterizing several structures.

• Our structure of interest is SRR resonators inside a cutoff waveguide
## Summary of Previous Results

<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>Resonant Frequency (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass-band, simulation</td>
<td>2.73</td>
</tr>
<tr>
<td>Pass-band, experiment</td>
<td>2.84</td>
</tr>
<tr>
<td>Stop-band, simulation</td>
<td>2.8</td>
</tr>
<tr>
<td>Stop-band, experiment</td>
<td>2.79</td>
</tr>
<tr>
<td>Simulated Infinite planar array</td>
<td>2.76</td>
</tr>
</tbody>
</table>

**Resonant Frequencies of different systems**

**Split-Ring Resonator:**
- 5 mm large outer ring radius
- 4.1 mm small ring outer radius
- 0.6 mm ring width
- 0.3 mm gap between rings
- 0.5 mm break in each ring
- 12 mm³ cubic unit cell

![SRR cards in cutoff Waveguide](image1.png)

![SRR cell](image2.png)
Summary of Previous Results

Metamaterial Time Behavior Experiment (MTBX)
Summary of Previous Results

Bandpass system

```
Bandpass system S11
```

Bandstop system

```
Band-Stop system S21 (experimental)
```

```
Bandstop system S21 (experimental)
```
Summary of Previous Results

**Bandstop System**
- Input/reflected signal for the bandstop system at 2.8GHz

**Bandpass System**
- Input/reflected signal for the bandpass system at 2.73GHz
Summary of Previous Results

- The time difference between the blue in (a) and green curves represents the propagation time in the waveguide.
- Linear initial behavior is clear in the control case while in the SRR-system is non linear and shows 3 slopes.
- Red curve shows an exponential roll-off similar to the one in the green curve → we attribute it to the propagation time in the WG.
- The first time constant in the red curve is attributed to the filling time of the SRRs.

<table>
<thead>
<tr>
<th>System</th>
<th>Delay</th>
<th>Rise time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without Waveguide</td>
<td>20 ns</td>
<td>15 ns</td>
</tr>
<tr>
<td>Empty Waveguide</td>
<td>26 ns</td>
<td>24 ns</td>
</tr>
<tr>
<td>SRR Pass Band</td>
<td>34 ns</td>
<td>40 ns</td>
</tr>
</tbody>
</table>
Modeling of the SRR filled WG System as an LTI System

- 1\textsuperscript{st} passband (narrow) corresponding to the SRR resonance
- 2\textsuperscript{nd} passband corresponding to the WG 1\textsuperscript{st} TE10 passband
Modeling of the SRR filled WG system as an LTI system

Motivation:
- give more insight on metamaterial behavior → Will be useful to understand the time domain behavior (e.g.: HPM and pulsed power applications)
- a simple model for metamaterial structure design
- control certain parameters to achieve certain performances

• The capacitance and inductance of the SRR have closed form expressions
Modeling of the SRR filled WG system as an LTI system

- Metamaterial structures have been modeled before as TLs with distributed elements

Left-Handed Transmission line

- Negative phase velocity and positive group velocity

\[
\frac{-1}{\omega C} = \mu \omega \rightarrow \mu(\omega) = \frac{-1}{\omega^2 C} \quad \text{and} \quad \frac{-1}{\omega L} = \varepsilon \omega \rightarrow \varepsilon(\omega) = \frac{-1}{\omega^2 L}
\]

Composite Right/Left-Handed Transmission line

- Presence of a Stop band in the freq. range (unbalanced case)
- Phase velocity depends on frequency:
  - <0 before the stopband
  - >0 above the stop band
Modeling of the SRR filled WG system as an LTI system

• Waveguides below cutoff have been as well modeled in the early 70ies (Craven) in order to design Evanescent mode WG Bandpass filters

\[ \gamma = \frac{2\pi}{\lambda} \sqrt{\left( \frac{\lambda}{\lambda_c} \right)^2 - 1}, \]

where \( \gamma \): propagation constant

\[ X_0 = \frac{120 \pi b}{a} \sqrt{\left( \frac{\lambda}{\lambda_c} \right)^2 - 1}, \]

where \( a, b \): broad/narrow dimension of the WG

\( \lambda_c \): cutoff frequency of the waveguide

Considerations:
As for all Metamaterial studies, in order to consider the medium homogenous, we consider that the unit cell size (corresponding to one SRR) is at least 10x smaller than the incident wavelength

Then we can assume that :

\[ \tanh(\gamma l) \cong \gamma l \]

\[ L_1 = \frac{jX_0 \tan(\gamma l)}{w} \cong \frac{X_0 \gamma l}{2w} = \frac{120 \pi b}{2a} \sqrt{\left( \frac{\lambda}{\lambda_c} \right)^2 - 1} \frac{2\pi}{2\lambda \sqrt{\left( \frac{\lambda}{\lambda_c} \right)^2 - 1}} \frac{l}{2\pi C} = \frac{120\pi bl}{2aC} \]

\[ L_2 || L_0 \cong L_0 \]

(a), (b), and (c) Equivalent circuits of evanescent mode waveguide. (d) Lumped equivalent circuit of evanescent mode waveguide filter.
Similarities between previous results and model

- 14 cell (pass-band system):
  - The TL cutoff frequency is 3.35GHz (for PEC-vacuum SRR with same size)
  - The calculated resonance frequency of one SRR is 3.41 GHz
  - Epsilon is negative
  - Mu is negative below the TL cutoff and positive above it
Similarities between previous results and model

- Group velocity is positive
- Phase velocity is negative
→ Phase velocity and group velocity are antiparallel

S-parameters of CST simulation for PEC-Vacuum SRRs

We are interested in the first passband that is below the cutoff of the waveguide

- Maximum Power transmitted is at 3.21 GHz where mu and epsilon are blow zero
- The shift in resonance in S21 is due to the impedance mismatch with the 50 Ohm system
Similarities between previous results and model

We are considering two different models in calculating the S-parameters of a multiple cell structure:

- Model 1:

![Model 1 diagram]

\[ L_1/2 + L_2 + L_0 = 1 \text{ cell} \]

The model seems to correspond to the CST simulation only at the maximum of transmission:

- The first passband in the CST-simulation is narrower than the passband in the LTI model
- The phase of the S11 between 3-3.5 GHz is similar to the phase of the S11 in the model.
- However looking at the S21 phase, the structure’s behavior appears to be more complex than the phase behavior of an LTI filter
Similarities between previous results and model

- Model 2:

\[ \frac{L_1/2}{2C_0} - \frac{L_2}{L_0} - \frac{2C_0}{2C_0} = 1 \text{ cell} \]

- The first passband in the LTI Model 2 is broader than the passband in the CST-simulation.

- The phase of the S21 between 3-3.5 GHz in Model 2 is similar to the phase of the S21 in CST simulation.
Similarities between previous results and model

- Existence of exponential roll-off in both Models
- No excitation delay in Model 1
- Signal in Model 1 reaches steady-state faster

<table>
<thead>
<tr>
<th>CST Simulation</th>
<th>LTI model 1</th>
<th>LTI model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay of 1.25ns</td>
<td>Very short delay (approx. 0.1ns)</td>
<td>Delay of 1ns</td>
</tr>
<tr>
<td>at least 2 time constants</td>
<td>at least 2 time constants</td>
<td>3 time constants</td>
</tr>
<tr>
<td>The settling time of the signal is very slow (approx. 20ns)</td>
<td>The settling time of the signal is lower than 5ns</td>
<td>Settling time relatively long (approx. 15ns)</td>
</tr>
</tbody>
</table>
Similarities between previous results and model

Presence of backward wave:

- The output wave phase is leading the input wave phase in both experimental set-up and model
- CST simulation showing the propagation of the backward wave (auxiliary Slide)
Model Comparison with Genetic Algorithm

- We use genetic algorithm as a means of verification for the model:

  Given the S-parameters from the CST-Simulation and the Linear model number of elements, the algorithm is capable of looking for the value of elements which would converge to the CST S-parameters and compute the mean square error.

The genetic algorithm converges for the same model and number of cells to a resonance that corresponds to the CST simulation resonance.

→ This supports that the LTI model can approach the behavior of the MTM system.
Model Limits

• The MTM-structure is considered linear-time-invariant

• We can consider the validity of the model only around the resonance frequency:
  - The actual model cannot describe the whole system
  - The model fails to approach the phase of the MTM in waveguide structure

• The genetic algorithm and CST-simulation results show some agreement with the linear model approximation of the system. However, this does not give precise values for the filter elements
Conclusion

• We studied using simulation and measurements the time domain and excitation behavior of MTM structure

• We tried to approach the frequency/time domain MTM structure behavior using a simple linear filter model

• The presence of a backward wave was shown in the simulation, experiment and model

• CST simulation results and genetic algorithm converged for the model that we are using
Ongoing/Future work

• Improve the LTI model to be able to predict with more accuracy the MTM structure behavior

• Assuming the LTI model is accurate, we can use control theory by applying a state feedback - \( Kx \) in order to control the time-behavior of the system

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + D
\end{align*}
\]

\[
\begin{align*}
\dot{x} &= (A - BK)x \\
y &= Cx + D
\end{align*}
\]

• This would theoretically permit us to cancel the initial delay and to decrease the settling time of the system

→ Repercussions on the frequency-domain behavior
→ Investigate the physical meaning of the feedback and how to implement it
Thank You