New concepts in traveling wave tubes based on multiple transmission lines

Mohamed Othman¹, Filippo Capolino¹, Alex Figotin²

¹Department of Electrical Engineering and Computer Science
²Department of Mathematics

August 1, 2014
Outline

Part I
- Multiple Transmission Line (MTL) concept and extension of Pierce model
  - Uniform MTL – e-beam dispersion relation and modes

- Possible routes for gain enhancement in uniform finite traveling wave tubes (TWT)
  - Pierce parameters
  - Gain

Part II
- Periodic MTL TWT
  - High-pass-type circuit
  - Dispersion relation with wide band interaction
  - Frozen mode regime in finite MTL TWT

The goal is to determine whether an MTL can enhance the gain of TWTs
Multiple transmission line (MTL) is a generic concept that can model a variety of coupled EM guiding structures.

The purpose of utilizing MTL is to enhance wave/matter interaction, and introduce peculiar dispersion characteristics that cannot be attained using single transmission line circuits.

MTL involves rigorous mathematical formulation, since it deals with matrices that are not necessarily diagonalizable.

We address two cases:

- Uniform MTL TWT
- Periodic MTL TWT

Examples:
- Coupled Helices
- Coupled cavity TWT
Pierce theory: review

Pierce theory: simple, analytical theory still in widespread use today

- Ideal representation of electron beam as a fluid.
- Complex slow wave structure idealized as a simple one-dimensional transmission line (TL) described by two parameters: inductance ($L$) and capacitance ($C$).

Pierce theory: analysis

- Linearization
  \[ \rho = \rho_0 + \rho_b, \quad I = I_0 + I_b = u_0 \rho_0 + u_0 \rho_b + v \rho_0 \]
- Equation of motion & continuity equation
  \[ (\partial_z + u_0 \partial_z) v = -\eta E_z \Rightarrow (k - \beta_0)^2 I_b = -j \frac{\rho_0 \eta \beta_0}{u_0} E_z \]
- Transmission line equation
  \[ \partial_z V = -j \omega LI, \quad \partial_z I = -j \omega CV + i_s \]
- Dispersion: \( k \) is the modal propagating constant

\[
1 + \frac{k^2 \beta_c \beta_0}{(k - \beta_0)^2 (k^2 - \beta_c^2)} 2C^3 = 0 \quad \Rightarrow \quad k \approx \beta_0
\]

Varying as \( e^{j\omega t} e^{-jkz} \)

\[ G \approx -9.54 + 47.3CN \]

4 solutions for \( k \): 2 complex, 1 forward, 1 backward

Bunching, space charge waves

Extension of Pierce theory to MTL

Complex slow wave structures and real waveguides are better represented by Multiple transmission Lines (MTL) since most SWS or real waveguides naturally have multiple modes.

Need for more detailed formulation to extend Pierce theory to MTL.

Coupled helix waveguide traveling wave tube. This is an example of two coupled slow-wave structures interacting with a single electron beam.

MTL formulation
Consider a waveguide system with a uniform cross-section that is able to support fields in the form

\[
\mathbf{E}_t(\mathbf{r}) = \sum_{n=1}^{N} \mathbf{e}_n(\rho)V_n(z)
\]

\[
\mathbf{H}_t(\mathbf{r}) = \sum_{n=1}^{N} \mathbf{h}_n(\rho)I_n(z)
\]

Defining equivalent space charge voltage and currents

\[
I_b = \rho_b u_0 + \rho_0 u_b, \quad V_b = \frac{u_0 v_b}{\eta}
\]

We allow coupling between the modes, and we define a state vector

\[
\Psi(z) = \begin{bmatrix} V_1 & V_2 & \cdots & V_N & V_b & I_1 & I_2 & \cdots & I_N & I_b \end{bmatrix}^T
\]

The state vector defines the evolution of the EM waves/space charge waves along the z-direction.

Notations

• MTL described by inductance and capacitance matrices $L$ and $C$

• Assume that $L$ and $C$ are symmetric and positive-definite

• $V$ and $I$ vectors on MTL: $V = [V_1, V_2, ..., V_N]^T$ and $I = [I_1, I_2, ..., I_N]^T$

• Strength of each shunt current generator can be scaled $s = [s_1, s_2, ..., s_N]^T$

• Assume $E_z$ of $n^{th}$ TL is related only to its voltage: $E_{z,n} = -a_n \partial_z V_n$

• Describe how each TL affects beam dynamics: $a^T = [a_1, a_2, ..., a_N]$

• We assume $s = a \quad \beta_c$ is the cold structure modal wavenumber

• This needs to be revisited in future (possibility of exotic dispersions)

• Assume solutions varying as $e^{j\omega t} e^{-j k z}$

  • $k$ is complex propagation constant and $\omega$ is radian frequency
Analysis of MTL TWT

Basic formulation involving evolution equations for the system

\[ \partial_z V = -j\omega LI \]

\[ \partial_z I = -j\omega CV + i_s \]

- e-beam is seen as a current generator
- Provides power to the TL

Objectives

- Extraction of modal characteristics

  dispersion relation \( k - \omega \) where \( k \) is the modal wavenumber

- Evaluation of coupling parameters

  Coupling impedance, Pierce parameters

- Calculation of gain for finite size TWT
Evolution of space charge waves along the z-direction

\[ \partial_z V = -j \omega L I, \quad \partial_z I = -j \omega CV + i_s \]

- Assume beam induces current in every TL (shunt current generator)
- Use linearized beam equations like in Pierce theory

Need to find complex propagation constant $k$ of the resonant modes

Establish transverse resonance condition in terms of beam impedance $Y_b$ and MTL impedance $Y$

Transverse resonance condition

$$(Y + Y_b)V = 0 \quad \Rightarrow \quad \det(Y + Y_b) = 0$$

Dispersion relation

$$\det \left( k^2 1 - \omega^2 LC + \frac{\eta \rho_0 \beta_0^2 k^2}{(k - \beta_0)^2} Lsa^T \right) = 0$$

Condition for growing waves

$$\beta_0 \geq \max \left\{ \beta_{c,1}, \beta_{c,2}, ..., \beta_{c,N} \right\}$$

Uniform MTL: Growing waves

- Natural propagation constants of MTL: \( \{\beta_{c,1}, \beta_{c,2}, \ldots, \beta_{c,N}\} \)

- Growing wave solutions always exist if electron propagation constant satisfies

\[
\beta_0 \geq \max \left\{ \beta_{c,1}, \beta_{c,2}, \ldots, \beta_{c,N} \right\}
\]

Illustrative example: 2-TL system coupled to beam with \( \beta_{c,2} > \beta_{c,1} \)
Gain for 2 TL: Pierce model

We assume \( k = \beta_0 + \delta \) As in Pierce model

\[
C_p = \begin{bmatrix} C_{p1} & C_{p2} \end{bmatrix}^T, \quad C_{p1} = \frac{x_1}{(4V_0 / I_0)}, \quad C_{p2} = \frac{x_2}{(4V_0 / I_0)}
\]

Represent the coupling between TLs and the e-beam

\( x_1 \) and \( x_2 \) are the coupling impedances to the lines

Gain can be represented in terms of Pierce parameters

\[
G_{dB} = -\alpha_0 + \left( \alpha_1 C_{p1} + \alpha_2 C_{p2} \right) L
\]

Gain can be enhanced using MTL!

\( \alpha_0, \alpha_1, \alpha_2 \) are constants
Coupling parameters

Condition for synchronism
Trade off between Gain and Synchronism

Beam parameters: $V_0 = 6$ KV, $I_0 = 3$ A, $u_0 = 0.15c$

TL parameters: $C_{11} = C_{22} = 80$ pF, $L_{11} = L_{22} = 7 \mu$H
Uniform MTL TWT: remarks

- The dispersion relation is written as a polynomial

\[ F(k, \beta_0, \beta_{c,1}, \beta_{c,2}, \omega) = 0 \]

\( k \) has only one pair of complex roots

**Theorem:** For uniform MTL described by \( L \) and \( C \), there exists only one unique growing wave mode*

Is it possible to have more than one growing mode?

Summary for uniform MTL

Uniform MTL TWT concept:

- Developing the Pierce theory for more than one transmission line
- Finding the modes – dispersion characteristics:
- Calculating the gain for finite size structures: possible gain enhancement is observed
- Assessing the coupling parameters, especially impedance (matrix!): provide intuition on how to properly design the TWT

Following steps:

- Developing concrete boundary conditions for finite MTL TWT
- Designing real structures that support 2 modes such as coupled helixes or coupled ladder circuits

Next topic: Periodic circuit
In contrast with helix TWTs, CCTWTs operate at higher-average power levels and **higher frequencies** but with **smaller bandwidths** than helix TWTs.

The interaction structure in a CCTWT is composed of a series of cavities that are connected via slots and a beam tunnel.

The **voltage across the gaps** modulates the electron beam velocity.

Future designs will allow interaction with more than one mode.

---

Curnow, *IEEE T-MTT* (1965)

Periodic MTL – e-beam interaction

\[e^{j\omega t} e^{-jkz}\]

Beam 

\[X_{i,j}\] model interfaces between different waveguide cross-sections

All a.c. quantities: \[e^{j\omega t} e^{-jkz}\] and \(k = \beta - j\alpha\)


MTL high pass type CC-TWT circuit

High pass type periodic circuit

Unit cell composed of **THREE** MTL segments with different coupling

Transfer matrix of one unit cell

Dispersion relation is found

\[ T \Psi(0) = e^{-jkd} \Psi(0) \]

The dispersion may develop a **regular band edge (RBE)**
Or a **degenerate band edge (DBE)**

Cold structure dispersion + electron beam

\[ \Delta \omega \approx \Delta k^4 \text{ DBE} \]
\[ \Delta \omega \approx \Delta k^2 \text{ RBE} \]
Dispersion of coupled system

- **Wide band interaction**
  Interaction with two modes at two different frequencies creates a wide band region where amplification is possible

- **More than one growing wave**
  Unlike the uniform MTL, here it is possible to have two growing waves **simultaneously**!

![Graph showing wide band interaction and multiple growing waves](image)

- Blue: exponential growing mode
- Green: exponential growing mode
- Black: exponential decaying mode
- Red: real $k$ mode

**Wide band amplification**

Only forward modes
With positive real $k$

Two possible growing waves!
Interaction near a degenerate band edge

- Growing waves can be observed in the vicinity of DBE
- We are investigating the backward wave excitation at that condition.

Interaction near the edge of transmission band

Time-Domain analysis!
Frozen wave amplifying regime

The unique transmission properties of Fabry-Perot resonators with DBE can be utilized.*

Conclusions

- Complex slow wave structures and real waveguides are better represented by MTLs.

- MTL brings a promise in enhancing the gain of TWT, through optimizing the coupling parameters.

- Periodic MTL offers phenomenological paradigm change since all modes can be amplified.

Future Research Directions

- Investigate wave amplification in finite uniform MTL TWT.

- Investigate finite periodic TWTs, excitations, and termination.

- Develop the theory of interaction near a regular band edge (RBE) and a degenerate band edge (DBE).

- Analyze TWT MTL in time domain including nonlinear effects (software development).
Finite Difference Time Domain (FDTD) analysis of MTL interaction with e-beam

Considering the following definitions

\[ V = [V_1, V_2, V_3, \ldots, V_N, V_b]^T, \quad a_b = [a, 0]^T \]
\[ I = [I_1, I_2, I_3, \ldots, I_N, I_b]^T, \quad s_b = [s_1, s_2, \ldots, s_N, 0]^T \]

Time dependent equations can be written as

\[
\begin{align*}
(A \partial_z + B \partial_t) V + u_0 \partial_z (D V) &= -(L_b \partial_t + R_b) I \\
(C_b \partial_t + G_b) V &= -(E \partial_z + F \partial_t) I + \partial_z (H I)
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\partial_z \\
\partial_t
\end{bmatrix}
\begin{pmatrix}
A + u_0 D & B \\
E - H & F
\end{pmatrix}
\begin{bmatrix}
V \\
I
\end{bmatrix}
&= \begin{bmatrix}
-(L_b \partial_t + R_b) & I \\
-(C_b \partial_t + G_b)
\end{bmatrix}
\end{align*}
\]

where

\[
\begin{align*}
A &= \begin{bmatrix}
I \\
0
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}, \quad L_b = \begin{bmatrix}
L \\
0 \\
0 \\
0
\end{bmatrix}, \quad R_b = \begin{bmatrix}
R \\
0 \\
0 \\
0
\end{bmatrix}, \quad D = a_b^T \begin{bmatrix}
I \\
0
\end{bmatrix}
\end{align*}
\]
\[
\begin{align*}
E &= \begin{bmatrix}
I \\
0
\end{bmatrix}, \quad F = \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}, \quad G_b = \begin{bmatrix}
G \\
0 \\
0 \\
0
\end{bmatrix}, \quad C_b = \begin{bmatrix}
C \\
0 \\
\eta \rho_0 \\
0 \\
-u_0
\end{bmatrix}, \quad H = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

Objective: investigate evolution of MTL TWT and study oscillations

* Collaboration with Mehdi Veysi, UC Irvine
Thank you