EXPLORING DEGENERATE BAND EDGE MODE IN HPM TRAVELING TUBE

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MAIN OBJECTIVES FOR THE FIRST YEAR

- Explore degenerate band edge (DBE) modes for multidimensional transmission lines and waveguides.

- DBE mode with alternating axial electric field.

- Transmission line model of TWT that can account for significant feature of the amplification.

- Suggested design of realistic waveguide for HPM TWT supporting DBE.
Electron super bunching as they move along $z$-axis with velocity $u$

A, B, and C are three different waveguide sections with distinct transverse anisotropy.

TWT with super amplification via the DBE mode.

Frozen DBE mode magnitude profile

RBE mode magnitude profile

Frozen mode axial electric field phase profile
FROZEN MODE REGIMES
Stationary points of the dispersion relation. Slow waves.

\[ \nu_g = \frac{\partial \omega}{\partial k} = 0, \quad \text{at } \omega = \omega_s = \omega(k_s). \]

1. Dramatic increase in density of modes.
2. Qualitative changes in the eigenmode structure (can lead to the frozen mode regime).

**Examples of stationary points:**

- Regular band edge (RBE): \( \omega - \omega_g \propto (k - k_g)^2 \), \( \nu_g \propto (k - k_g) \propto (\omega - \omega_g)^{1/2} \).
- Stationary inflection point (SIP): \( \omega - \omega_0 \propto (k - k_0)^3 \), \( \nu_g \propto (k - k_0)^2 \propto (\omega - \omega_0)^{2/3} \).
- Degenerate band edge (DBE): \( \omega - \omega_d \propto (k - k_d)^4 \), \( \nu_g \propto (k - k_d)^3 \propto (\omega - \omega_d)^{3/4} \).

Each stationary point is associated with slow wave, but there are some fundamental differences between these three cases.
BASIC CHARACTERISTIC OF THE FROZEN MODE REGIME

- The frozen mode regime is not a conventional resonance – it is not particularly sensitive to the shape and dimensions of the structure.
- The frozen mode regime is much more robust than a common resonance.
- The frozen mode regime persists even for relatively short pulses (bandwidth advantage).
SLOW WAVE RESONANCE

Slow-wave phenomena in bounded photonic crystals.
Cavity Resonator vs. Slow Wave Resonator
Examples of Plane-Parallel Open Resonators

Simplest uniform resonance cavity with metallic reflectors

Uniform resonance cavity with photonic reflectors (DBR)

Single mode photonic cavity

Slow wave photonic resonator (no reflectors needed)
EM energy density distribution at resonance frequency

\[ W(z) = \frac{1}{8\pi} \left[ \varepsilon E^2(z) + \mu H^2(z) \right] \]

Uniform (empty) resonance cavity:
Standing wave

\[ \text{const} \]

Regular slow wave resonance at a RBE:
Standing Bloch wave:

\[ D^2 \sin^2 \left( \frac{\pi}{D} \right) \]

Regular slow wave resonance at a RBE:
Standing Bloch wave:

\[ D^2 \sin^2 \left( \frac{2\pi}{D} \right) \]

Giant slow wave resonance at a DBE: NOT a standing Bloch wave

\[ D^4 \]

Poor confinement  Better confinement  Best confinement
Transmission band edge resonances near a RBE

Transmission dispersion of periodic stacks with different $N$. 
$\omega_g$ – the RBE frequency

Smoothed energy density distribution at frequency of the first resonance

$max (W) \propto W_i N^2$
Giant transmission band edge resonances near a DBE

Transmission dispersion of periodic stacks with different $N$. $\omega_d$ – the DBE frequency

Smoothed Field intensity distribution at frequency of first transmission resonance

$$\max(W) \propto W_1 N^4$$
Summary: RBE resonator vs. DBE resonator

Regular Band Edge: \[ \omega \approx \omega_g - \frac{a_2}{2} \left(k - k_g\right)^2 : \]
\[ \max(W) \propto W_I \left(\frac{N}{m}\right)^2 \]

Degenerate Band Edge: \[ \omega \approx \omega_d - \frac{a_4}{4} \left(k - k_d\right)^4 : \]
\[ \max(W) \propto W_I \left(\frac{N}{m}\right)^4 \]
Example: Slow-wave cavity resonance in periodic stacks composed of different number $N$ of unit cells.

Energy density distribution inside photonic crystal at frequency of slow wave resonance

- Regular Band Edge: $\max(W) \propto W_r N^2$
- Degenerate Band Edge: $\max(W) \propto W_r N^4$

A DBE slow-wave resonator composed of $N$ layers performs similar to a standard RBE resonator composed of $N^2$ layers, which implies a huge size reduction.
The electric field in periodic structures (periodic except for an inter-element phase shift):

\[ E(r + d\hat{z}, k_z) = E(r, k_z) e^{ik_z d} \]

A mode is expressed in term of Fourier series expansion, and thus represented as the superposition of Floquet spatial harmonics

\[ E^{\text{mode}}(r, k_z) = \sum_{p=-\infty}^{\infty} e^{ik_{z,p} z} e_p^{\text{mode}}(x, y, k_z) \]

\[ k_{z,p} = k_z + 2\pi p / d \]

\[ k_{z,p} = \beta_{z,p} + i\alpha_z \]
Physical modes for coupling

- **Forward/Backward**
  \[ k_{z,p} = \beta_{z,p} + i\alpha_z \Rightarrow \begin{cases} \beta_{z,p}\alpha_z > 0 & \text{Forward waves} \\ \beta_{z,p}\alpha_z < 0 & \text{Backward waves} \end{cases} \]

- **Slow/Fast (coupling with field produced by electron bunches)**

  Slow Mode: *all* its Floquet wavenumbers are outside the “visible” region, or
  \[ |\beta_{z,p}| > k \]

  Fast Mode: mode has *at least one* Floquet wavenumber within the “visible” region, or
  \[ |\beta_{z,p}| < k \]
Physical waves in open periodic structures

<table>
<thead>
<tr>
<th>Wave Type</th>
<th>Condition</th>
<th>Note</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Forward Wave</strong></td>
<td>$\beta_{z,p} \alpha_z &gt; 0$</td>
<td>(A)</td>
<td>$\beta_{z,p} \alpha_z &gt; 0$</td>
<td>(B)</td>
</tr>
<tr>
<td><strong>Backward Wave</strong></td>
<td>$\beta_{z,p} \alpha_z &lt; 0$</td>
<td>(A)</td>
<td>$\beta_{z,p} \alpha_z &gt; 0$</td>
<td>(B)</td>
</tr>
<tr>
<td><strong>Slow Wave</strong></td>
<td>$</td>
<td>\beta_{z,p}</td>
<td>&gt; k$</td>
<td>(proper, bound)</td>
</tr>
<tr>
<td></td>
<td>$\alpha_{\rho,p} &gt; 0$</td>
<td></td>
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<td></td>
</tr>
<tr>
<td><strong>Fast Wave</strong></td>
<td>$</td>
<td>\beta_{z,p}</td>
<td>&lt; k$</td>
<td>(improper, leaky)</td>
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The theory is complicated, but it can be summarized as:

$$k_{z,p} = \beta_{z,p} + i\alpha_z$$

$$\beta_{z,p} = k_z + \frac{2\pi p}{d}$$
Methods for complex mode calculations

Peculiar modes investigated here need some fine determination:

- complex wavenumber or complex frequency descriptions
- pairing of modes (long discussion in literature)
- spectral points with vanishing derivative
- time domain description of polarization

**Methods:**

- Green’s function methods, combined with method of moments (MoM)
- Mode matching (field expansions)
- Commercial software is not able to determine complex modes, but it can be combined with properties of complex modes (i.e., moving around constraints of commercial software, HFSS, CST, FEKO, NEC)
- Analytic and physical properties
Points to be developed

- Field in periodic structures
- Complex modes in periodic structures
- Peculiar spectral points (RBE, SIP, DBE)
- Possible structures exhibiting peculiar points
- Excitation of complex modes in periodic structures and in truncated periodic structures
- Coupling of modes with fields produced by electron bunches
- Understanding complex modes in the time domain, including polarization evolution
Modes

- Waveguide with elliptical sections

The elliptical cross sections may act as anisotropic sections

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<th>Ratio of Third Derivative Zero Crossing</th>
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<tr>
<td>1</td>
<td>None</td>
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</tr>
<tr>
<td>2</td>
<td>2.85</td>
<td>2.90</td>
</tr>
<tr>
<td>3</td>
<td>2.90</td>
<td>3.00</td>
</tr>
<tr>
<td>4</td>
<td>2.10</td>
<td>2.60 and 2.80</td>
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<tr>
<td>5</td>
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Analyzing Modes

Vanishing derivatives (up to the third one)

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Second Derivative of Omega with Respect to Beta for Several Modes