Analysis of Quadrupole Focusing Lattices for Electron Beam Transport in Traveling-Wave Tubes

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***Abstract*—Analysis of quadrupole focusing lattices for high-frequency TWTs is presented. This work is motivated by recent work performed at the Naval Research Laboratory which demonstrated an advantageous case for strong focusing employing a Halbach quadrupole lattice. Using realistic PMQ field cancellation the advantage of using PMQ to transport higher current densities than PPM disappears, other advantages for employing quadrupole focusing remain.**

***Index Terms*—TWT; quadrupole strong focusing; permanent magnet; beam transport.**

I. Introduction

NOVEL millimeter wave vacuum electronic devices for communications, imaging, and radar applications require higher performance devices than are currently available. In this regime one device of interest is the traveling wave tube (TWT). TWTs have shown that they are robust, reliable, and have the potential to be scaled for very high frequencies. However, at high frequencies the device dimensions become minute and it becomes a challenge to obtain sufficient power. Beam voltage is limited by the necessity to avoid electric-field breakdown in the gun region, and by the desire to keep the power modulators to drive the devices compact. It is therefore in our interest to keep beam voltages low. To achieve sufficient power with lower voltages we must increase the beam current. Sheet beams are proposed as a solution to obtaining more beam current, but sheet beams, due to their dual-plan symmetry, cannot be focused using the compact focusing method of choice: standard permanent periodic magnet (PPM) focusing lattices. To avoid the use of a bulky solenoid or a non-standard PPM geometry, permanent magnet quadrupole (PMQ) lattices are proposed for compact focusing of sheet beams. PMQs have long been used by the accelerator community to focus high energy, emittance dominated beams. With the exception of [1], [2], there has not, however, been much work applying PMQs to focusing lower energy space-charge dominated beams appropriate for use in vacuum electronic devices such as TWTs. Quadrupole magnets have also been used extensively to form sheet beams, but rarely employed to transport them.

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The present research aims to provide a thorough analysis of quadrupole focusing for TWT beams. The analysis presented employs pencil beams, but in principle can be extended to the focusing of sheet beams simply by varying the spaces between the focus and defocus magnetic lenses. Previous work [1] demonstrated a valuable volume/weight advantage for focusing a pencil beam using PMQ over PPM, demonstrating a reduction of approximately 5 times in the required magnetic material to focus a comparable beam. Another important advantage of using PMQ focusing is the spacing between magnets in the lattices. In recent work studying metamaterial slow wave structures for TWTs, these spacings are facilitating the insertion of ports to allow for optical access to study the effect of the electron beam and resonant fields on the metamaterials [3].

Some results were presented at the 2013 International Vacuum Electronics Conference in Paris, France [4]. In this paper we expand the results to apply to various beam energies of interest and we contrast the maximum transportable current density with results from previous work. By using a more realistic magnet model, we are better able to determine the maximum transportable current density per lattice period for a quadrupole focusing lattice employing a 16-piece Halbach quadrupole model.

The remainder of this article is organized as follows. Section II discusses the formerly published analysis of maximum transportable current density for PPM and PMQ lattices and discusses the importance of including fringe fields in that estimation. Section III discusses the PMQ fringe field analysis that was performed and studied analytically and with simulations. Section IV explains the particle tracking work performed. In Section V, we present a new estimation for maximum transportable current density including the fringe-fields of the quadrupole magnets. The conclusions are presented in Section VI.

II. Work Presented

Previous work by researchers at NRL [1] presented a comparison of PPM vs. PMQ focusing for maximum transportable current as a function of lattice period. The estimation of maximum transportable current density follows from their equation 6 and is reproduced here as (1)

$$I_{PMQmax} = \frac{(\sigma_0 a_0 \beta\gamma)^2}{4L^2}[\frac{2}{I_0\beta\gamma} + \frac{\epsilon_{n1}^2}{I_1 a_0^2}]^{-1}, \qquad (1)$$

where β, γ are the relativistic constants, a_0 is the mean beam radius, σ_0 is the zero-current phase advance, L is the lattice

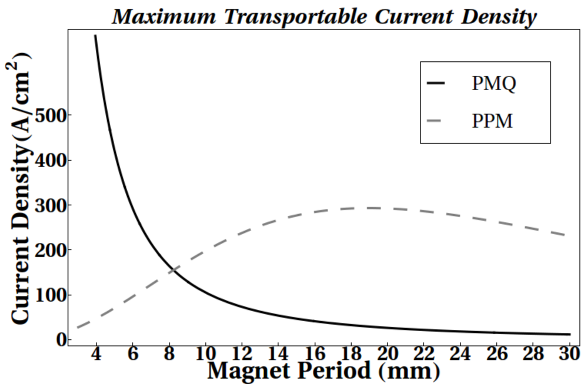

Fig. 1. Maximum transportable current density per lattice period from (1), and (2) motivated by [1].

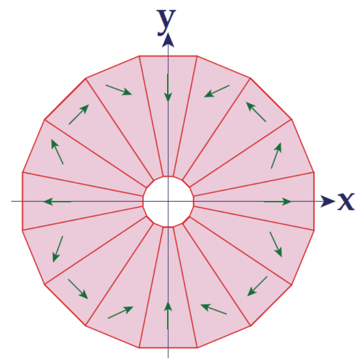

Fig. 2. Magnitization vectors of the 16-piece Halbach quadrupole magnet [6].

period, ϵ_{n1} is the normalized emittance for a gun producing current I_1, and I_0 is 17.1 kA.

The estimation of maximum transportable current for PPM focused beams follows directly the method presented in [1]. It is determined by equating the on-axis magnetic field estimation to the Brillouin field on axis and solving for maximum current, assuming balanced magnetic and space charge forces. This is presented as (2) below, where B_r is the magnetic field necessary for Brillouin flow, V_k is the beam energy, r_i, r_o, are the inner and outer magnet radii respectively. Note that the PPM current cannot be increased indefinitely by making the period, L, larger, but is subject to stability criteria. [5]

$$I_{PPMmax} = \left(\frac{a_o {V_k}^{1/4}}{8.32 * 10^{-4}}\right)^2 \frac{B_r^2 L^2}{32} \left(\frac{1}{\sqrt{r_i^2 + \left(\frac{L}{4}\right)^2}} - \frac{1}{\sqrt{r_o^2 + \left(\frac{L}{4}\right)^2}}\right)^2 \quad (2)$$

In Fig. 1 we have recreated Fig. 7 from [1] but normalized for current density. This figure represents a 16 keV beam with an inner magnet radius of 4 mm. This graph shows a strong advantage for transporting higher current density pencil beams in cases where a shorter lattice period is appropriate. However, (1) depends on the assumption of a hard-edged quadrupole model which, we will show, does not apply well to quadrupole lattices of relevant dimensions for focusing space-charge dominated beams. Relevant lattice dimensions for this purpose necessarily place the quadrupole magnets close together such that the fringe fields significantly overlap which cannot be neglected.

A detailed analysis of the magnet lattices has been performed along with optimization of a variety of lattices with varying periodicity which allows us to develop a more accurate estimation of maximum current density transportable as a function of magnet dimensions and periodicity.

A detailed study of the magnetic field profile produced by PMQ lattices was performed and contrasted with the magnetic profiles of PPM lattices for the purpose of transporting high current density electron beams. It was found that the root mean square (rms) magnetic field of the PPM lattice suffers severe reduction in field strength as the lattice period decreases, giving an optimum lattice period for maximum current density transport. A PMQs main field components are of higher order, and as such the field profiles are less prone to reduction as the lattice period decreases, but do still reduce. Reduction of PMQ field strength was studied as a function both of the lattice period and the magnet length.

III. PMQ Field Analysis

Halbach derived an expression for the fringing magnetic fields for the 16-piece semi-infinite quadrupole magnet [6], [7]. The 16-piece Halbach quadrupole, see Fig. 2 for magnetization vectors, is chosen to have a higher gradient on axis than the standard 4-piece quadrupole magnet [8]. By modifying his expression to represent a finite magnet and using the principle of superposition, we develop an analytic expression for the gradient of the magnetic field for a lattice of $n+1$ quadrupole magnets [9]:

$$\frac{dB}{dx} = \sum_{i=1}^{n} G\mathcal{F}[i, L, l] - \sum_{i=2}^{n+1} G\mathcal{F}[i, L, l] \quad (3)$$

where

$$\mathcal{F} = F\left[-i\left(\frac{L}{2} - l\right) + L - \frac{l}{2}\right] - F\left[-i\left(\frac{L}{2} - l\right) + L + \frac{l}{2}\right] \quad (4)$$

and

$$F = \frac{1}{2} - \frac{z}{16}\left(\frac{1}{r_i} + \frac{1}{r_o}\right)\left(\frac{{v_i}^2 {v_o}^2}{v_i + v_o}\left({v_i}^2 + {v_o}^2 + 4 + 8 v_i v_o\right)\right) \quad (5)$$

and

$$v_{i,o} = \frac{1}{\sqrt{1 + \frac{z}{r_{i,o}}^2}}. \quad (6)$$

The constant G is defined by:

$$G = 2B_{pole}\left(\frac{1}{r_i} - \frac{1}{r_o}\right) \operatorname{sinc}\left(\frac{3\pi}{M}\right), \quad (7)$$

where M is the number of sections in the quadrupole and B_{pole} is the magnetic field at the pole tip.

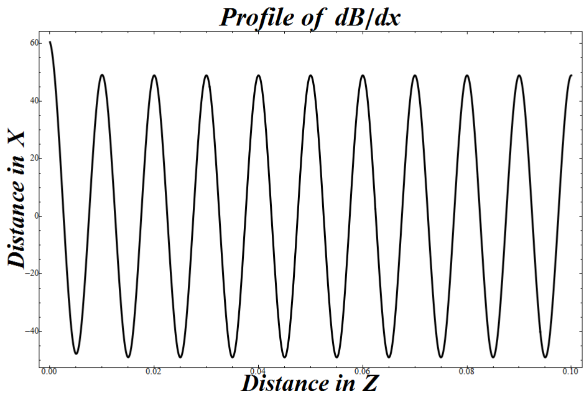

Fig. 3. PMQ Field Profile from 3.

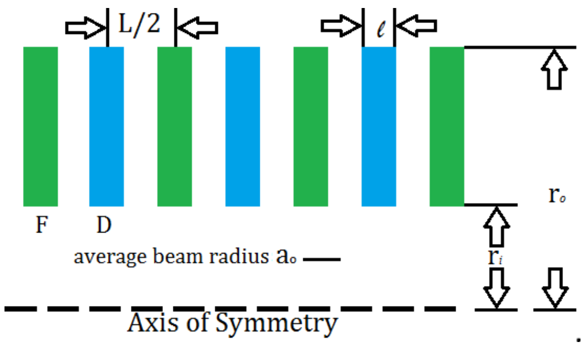

Fig. 4. Quadrupole lattice geometry.

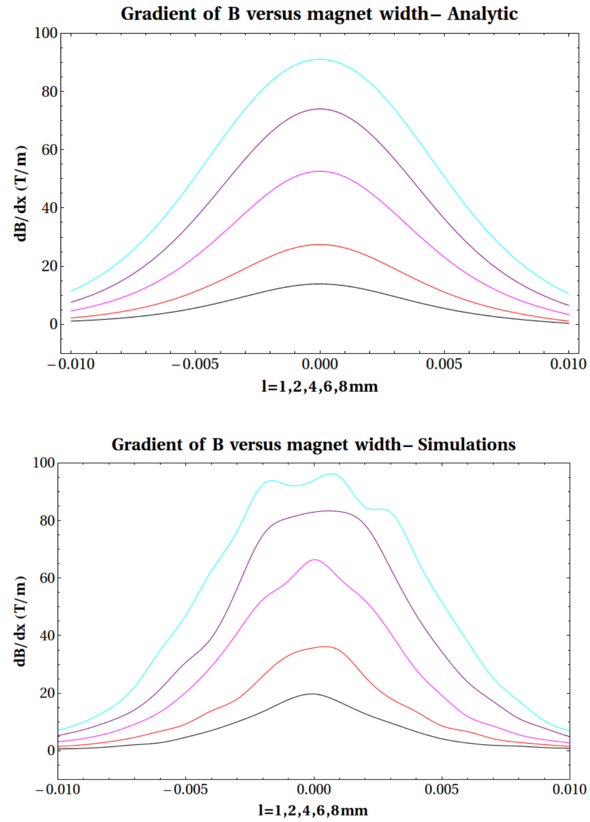

Fig. 5. Analytic quadrupole model, above, simulated quadrupole model, below.

The gradient of the magnetic field is shown for the x-z plane in Fig. 3 for 9.5 magnet periods, or 19 quadrupoles. The quadrupole lattice geometry and representative symbols are shown in Fig. 4, where F represents a focusing lens, and D a defocusing lens.

For verification of this analytic expression, we constructed the same geometric magnet model in the Ansoft code Maxwell [10]. The Maxwell model accounts for the complicated physics of the permanent magnet interactions. The magnetic material was chosen as $SmCo_{28}$, and the magnetic pole field was calculated as 0.9 T. The simulations agreed with the analytic data within 5% for a variety of magnet dimensions. The analytic model was used for the rest of this work. A comparison of the analytic model to the Maxwell model for various magnet widths, l, is shown in Fig. 5.

IV. Particle Tracking in PMQ

Single particle-tracking was performed for both PPM and PMQ focusing lattices to ensure stable transport. An envelope code, including space-charge, was developed with a full fringe field model of the Halbach 16-piece quadrupole magnets. Utilizing the envelope code, optimization of the PMQ lattice was performed for maximum current density transport for a given beam energy and appropriate magnet dimensions.

For this work, we have chosen lattice parameters appropriate for transporting a beam in a 30 GHz coupled-cavity TWT. The beam energy is varied from 16 keV to 50 keV and the mean beam radius is 0.5 mm.

A. Single Particle Tracking

Single particle tracking is performed to ensure that the beam remains stable according to the Mathieu stability conditions. The beam is determined to be stable when the zero-current phase advance σ_0 is less than 90 degrees [11]. The equations of motion (8) and (9) were solved with the full magnetic field profile of the quadrupole lattice from (3) to analyze single particle tracking. The equations were solved to give the position of the particle in the x–z and the y–z planes:

$$x''[s] + \kappa_x x[s] = 0 \tag{8}$$

$$y''[s] + \kappa_y y[s] = 0 \tag{9}$$

where

$$\kappa_x = \frac{e_{\text{charge}} \frac{dB}{dx}[s]}{\gamma_{rel} e_{mass} v \beta} \tag{10}$$

and $\kappa_x = -\kappa_y$.

These equations were solved using the standard differential equation solver in *Mathematica* [12], and the periodic solution for the x–z plane is shown in Fig. 6. A sinusoidal curve fit was used to determine the period of the particle trajectory.

The periodicity is then used to determine the phase advance per lens σ_0. To maintain a stable beam while transporting maximum current, σ_0 is kept as close to 90 degrees as possible without exceeding it. For comparison with theory, a lattice with no overlapping fringe-fields was selected and the phase advance determined by envelope simulations was compared to

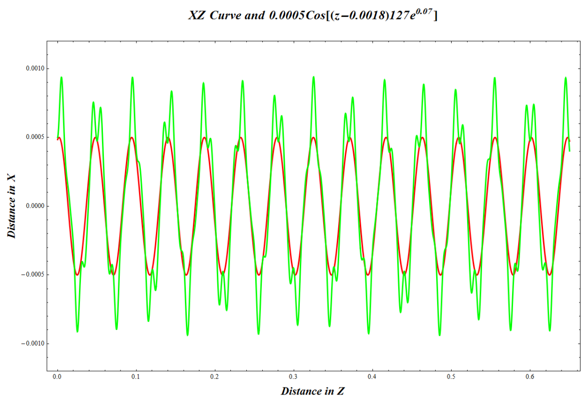

Fig. 6. The particle trajectory in the x-z plane, green, and a period fitting function, red.

the phase advance determined analytically using the hard-edge quadrupole approximation from equation (3.34) in [11]:

$$\sigma_0 = \cos^{-1}[\cos\theta\cosh\theta + \frac{L\theta}{l_z} * (cos\theta\sinh\theta - \sin\theta\cosh\theta)], \tag{11}$$

where θ is a well known function of κ. For the lattice shown in Fig. 4, with $l = 1.9$ mm and $\frac{L}{2} - l = 5$ mm, the phase advance as calculated using Eq. (11) is 58.7 degrees, and the phase advance derived from the particle tracking is 64 degrees. This is within the standard error for the hard-edge quadrupole model.

B. Space-Charge Effects

The maximum transportable current density can be found by including space charge effects in the single particle tracking calculation. The equations of motion then become the envelope equations. Since the space-charge term is a function of current, we determine the maximum current density transportable for a given PMQ lattice by incrementally increasing the current density until the depressed phase advance, the phase advance of the beam envelope with space-charge, goes to zero.

Space-charge effects were accounted for by including the space charge term [13] in equations (8),(9). This gives us these two coupled non-linear second order differential equations:

$$x''[s] + \kappa_x x[s] - \frac{2K_0}{x[s]+y[s]} = 0 \tag{12}$$

and

$$y''[s] + \kappa_y y[s] - \frac{2K_0}{x[s]+y[s]} = 0, \tag{13}$$

where K_0 is the generalized perveance and a function of beam current. Solving (12), (13) allows us to calculate the depressed phase advance. When the depressed phase advance goes to zero degrees, the beam is matched and the current density transport is maximized. Space-charge unmatched and matched beam results from the code are shown in Fig. 7. The small oscillations represent the particles moving through a single set of quadrupole magnets showing the defocusing and refocusing of the beam-edge, the periodicity matching that of the magnet lattice.

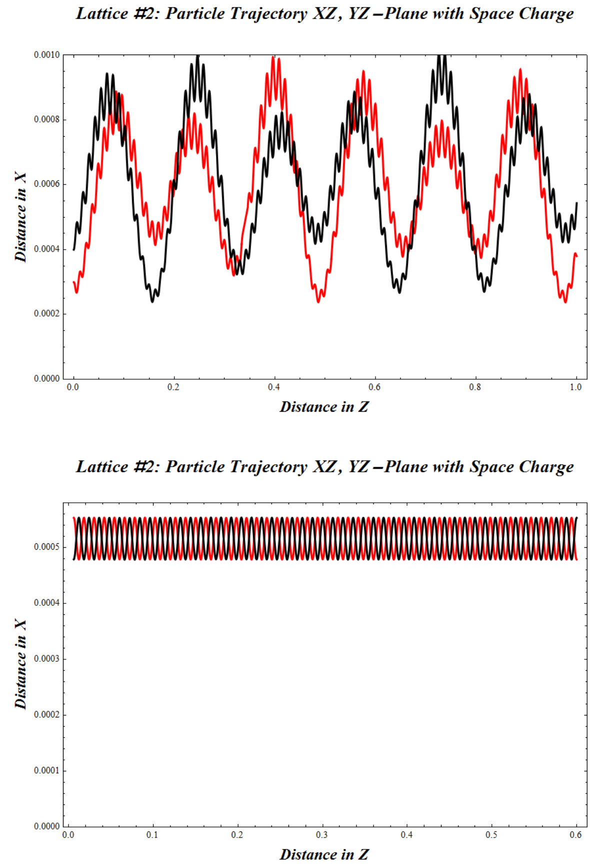

Fig. 7. The x–z particle trajectory, black, and the y–z particle trajectory, red. Above, unmatched beam, below, matched beam.

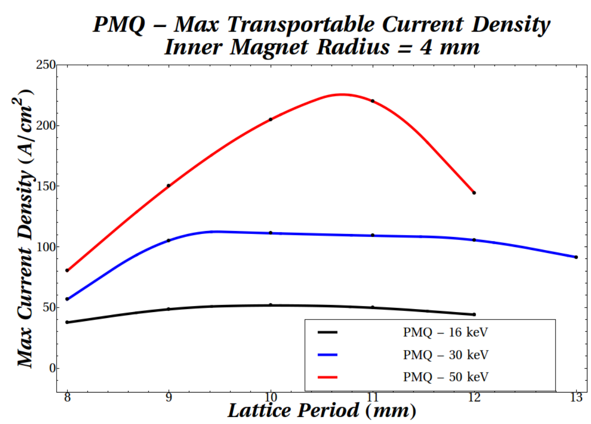

Fig. 8. Maximum transportable current density optimized per lattice period for various beam energies.

V. Maximum Transportable Current Density

The maximum transportable current density was determined for the limiting factors of beam energy and inner magnet radius. The outer radius of the magnets can be made as large as needed to reach the asymptotic value of magnetic field intensity without being unreasonably large. As such, the outer magnet radius is not a limiting factor in transportable current density. For an inner magnet radius of 4 mm, the current density as a function of periodicity is optimized. Optimization was performed by varying the magnet width l and the space between magnets in the single-particle-tracking code to obtain the zero-current phase advance as close to 90 degrees as possible. As the lattice period becomes small, the beam can not achieve 90 degrees phase advance with any magnet thickness, resulting in the drop off in maximum current density seen in Fig. 8, but not seen in Fig. 1. In Fig. 8, it is the black line representing the 16 keV beam which is comparable to Fig. 1. For a variety of reasonable beam energies, the results for maximum transportable current density are presented in Fig. 8.

VI. Conclusions

The maximum transportable current density using a PMQ lattice is not as high as initially expected, and may not be an advantage over using traditional PPM focusing in the regimes where these magnet dimensions are appropriate. To scale the magnet inner radius larger results in low field values which can only transport limited current and to scale the inner magnet radius smaller results in field values too large to produce stable transport. However, the transportable current density is still sufficient to transport most beams of interest to high-frequency TWT devices, and is able to do so using less magnetic material and with less weight than current PPM lattices.

Detailed analysis and optimization of PMQ lattices including overlapping fringe-field effects has been performed for pencil beams and is highly transferable to the compact transport of sheet beams by varying the drift sections between magnets. The advantage of PMQ focusing over PPM focusing remains size, weight, and empty spaces in the lattice appropriate for diagnostics. The spacing between magnets in the lattices provided by PMQ focusing is useful for diagnostic access to novel TWT structures such as metamaterial interaction circuits.

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