Generalized Energy Demodulation for Large Frequency Deviations and Wideband Signals

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***Abstract*—The Teager–Kaiser energy operator and the related energy separation algorithm (ESA) find numerous applications in various problems related to monocomponent AM-FM demodulation. This energy-operator-based approach, however, applies only to signals with narrowband frequency content, i.e., the information bandwidth and frequency deviation about the carrier are small relative to the carrier frequency. For signals with large frequency deviation, modulation indices, this approximation fails, and the ESA incurs large frequency/amplitude demodulation errors. In this letter, we develop a generalized energy-operator-based approach that uses frequency transformations derived from multirate operations such as decimation/interpolation and heterodyning. This generalized approach is shown to produce significant reduction of the demodulation error over the conventional ESA, particularly where the modulation index or frequency deviation is large.**

***Index Terms*—Energy operator, frequency and amplitude demodulation, heterodyning, large frequency deviations, multirate frequency transformations, wideband signals.**

I. Introduction

MONOCOMPONENT AM-FM signals are time-varying sinusoids[1] of the form

$$x(t) = a(t)\cos\left(\int_{-\infty}^{t}\omega_i(\tau)d\tau + \theta\right)$$

where the *instantaneous frequency* (IF) and *instantaneous amplitude* (IA) of the signal are given by

$$\omega_i(t) = \omega_c + \omega_m q_i(t) \quad a(t) = A(1 + \kappa_a m(t))$$

where $q_i(t)$ is the normalized frequency message signal and $m(t)$ is the normalized AM information signal. Specifically, for sinusoidal FM modulation, the IF signal is of the form

$$\omega_i(t) = \omega_c + \omega_m \cos(\omega_f t + \theta).$$

The FM modulation index of the signal in this case is defined via the ratio $\beta = \omega_f/\omega_m$. When $\beta < 1$, the FM signal is narrowband, and when $\beta > 1$, it is wideband. The carrier -to-information-bandwidth ratio (CR/IB) and carrier-to-frequency-deviation ratio (CR/FD) parameters of the signal [7] are defined via the ratios

$$\frac{\mathrm{CR}}{\mathrm{IB}} = \frac{\omega_c}{\omega_f} \quad \frac{\mathrm{CR}}{\mathrm{FD}} = \frac{\omega_c}{\omega_m}. \tag{1}$$

These signals find numerous applications in speech processing, radar tracking problems, and biomedical applications [7]. The nonlinear Teager–Kaiser energy operators for continuous and discrete cases are defined via

$$\Psi_c(x) = [\dot{x(t)}]^2 - x(t)\ddot{x}(t)$$
$$\Psi_d(x) = (x[n])^2 - x[n-1]x[n+1] \tag{2}$$

where the dot denotes time derivative. When the signal $x(t)$ corresponds to the displacement in a simple harmonic oscillator, the output of the energy operator corresponds to the normalized energy of the oscillator. When the amplitude and frequency are time-varying, this operator in either case tracks the energy of the source that produced the signal $x(t)$. The conventional *energy separation algorithm* (ESA) uses the Teager–Kaiser energy operator to separate amplitude and frequency modulations

$$\omega_i(t) \approx \sqrt{\frac{\Psi_c(\dot{x})}{\Psi_c(x)}} \quad |a(t)| \approx \frac{\Psi_c(x)}{\sqrt{\Psi_c(\dot{x})}}. \tag{3}$$

The IF/IA demodulation errors of the ESA can be reduced further by applying simple binomial smoothing on the lowpass energy signals [5]. The ESA in particular is simple, efficient, and has excellent time resolution. For the IF/IA demodulation errors of the ESA to be small, the narrowband signal assumption must hold, i.e., the information bandwidth and the frequency deviation of the signal need to be small in comparison to the carrier frequency. This is not the case, however, when the signal of interest has a: 1) $\beta > 2$; 2) large information bandwidth as described in [4] and causes the ESA to incur larger IF/IA demodulation errors. Therein lies the motivation for a general approach that accommodates large frequency deviations or modulation indices. Previous attempts at demodulating large deviation FM signals incorporated aspects of backward-difference frequency discrimination, frequency feedback, and nonlinear prediction [10].

II. Generalized ESA Approach

The basic idea behind the energy demodulation algorithm for large frequency deviations (ESA-LDEV) is to apply frequency transformations to convert the large deviation signal into one with a smaller deviation. This is accomplished by compressing the monocomponent AM-FM signal in frequency by a factor R. For sinusoidal signals, scaling the signal in time produces a scaling in frequency, i.e., $y(t) = x(at) \longleftrightarrow 1/(|a|)X(\omega/a)$. Specifically, when $a = 1/R < 1$, this transformation corresponds to frequency compression, and when $|a| > 1$, this corresponds to a frequency expansion. While considering monocomponent AM-FM signals, the IF of the compressed signal $y(t)$

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[1]This model can accomodate CPFSK, MSK, GMSK, and other forms of digital phase modulation.

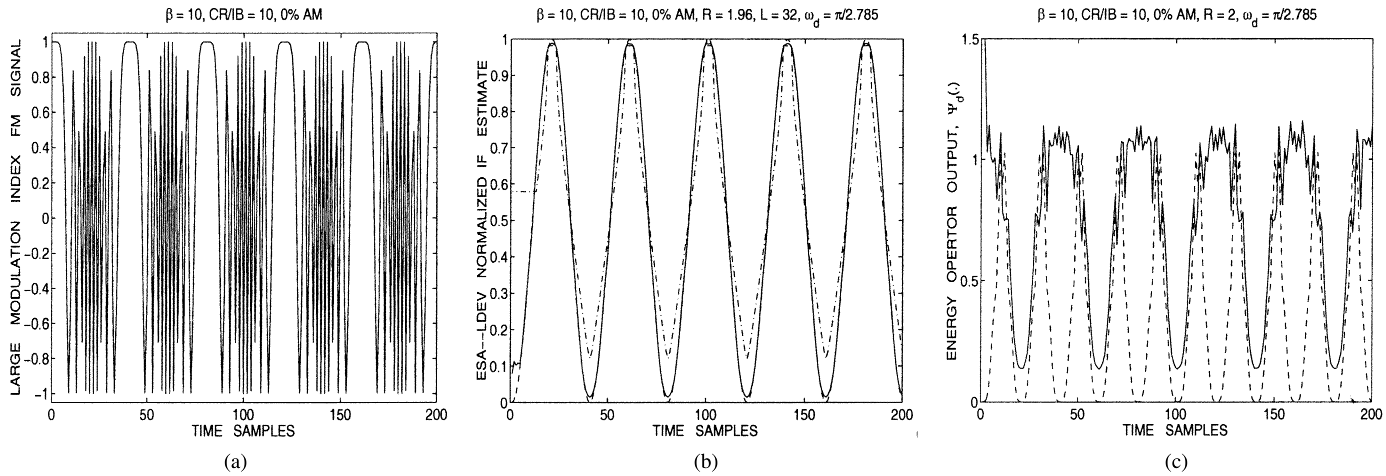


Fig. 1. AM-FM demodulation for wideband signals. (a) FM signal with $\beta = 10$. (b) IF estimates (solid line) of the ESA-LDEV approach for the same wideband example with a noninteger rate change factor $R = 1.96$, (dashed–dotted line) the conventional ESA, and (dashed line) actual IF. (c) Energy-operator output from (dashed line) original FM signal and (solid line) transformed signal depicting the effect of the multirate and heterodyne operations.

becomes a scaled version of the IF of the input signal $x(t)$ and is given by

$$\tilde{\omega}_i(t) = \frac{\omega_i\left(\frac{t}{R}\right)}{R} = \frac{\omega_c}{R} + \frac{\omega_m}{R} q_i\left(\frac{t}{R}\right). \tag{4}$$

Note that in the process the carrier frequency of the interpolated signal has also been scaled by the same factor, and this is undesirable, since we seek an input signal with a larger CR/FD ratio for the ESA to perform well. So, we translate the interpolated signal in frequency, i.e., frequency upshift the signal

$$y_{\text{ush}} = [y(t)\cos(w_d t)] * h_{\text{BP}}(t) \approx \left(\frac{1}{2}\right) a(t) \cos\left(\phi\left(\frac{t}{R}\right)\right)$$

where the $*$ operation denotes convolution and $h_{\text{BP}}(t)$ is the impulse response of the bandpass filter used to extract the desired high-frequency term in the modulation product. The resultant signal has a reduced message bandwidth but a higher carrier frequency of $\tilde{\omega}_c = \omega_c/R + \omega_d$. The ESA is then applied on this frequency upshifted signal to yield intermediate IF and IA estimates

$$\omega_i(t) \approx \sqrt{\frac{\Psi(\dot{y_{\text{ush}}})}{\Psi(y_{\text{ush}})}} \quad |a_i(t)| \approx \frac{\Psi(y_{\text{ush}})}{\sqrt{\Psi(\dot{y_{\text{ush}}})}}. \tag{5}$$

The final IF estimate of the proposed ESA-LDEV algorithm is obtained by translating, scaling, and expanding the ESA IF estimate

$$\omega_{\text{est}}(t) = R(\hat{\omega}_i(Rt) - \omega_d). \tag{6}$$

Frequency compression, therefore, serves the purpose of reducing the frequency deviation of $x(t)$ by a factor of R and compressing the IF, while still retaining the continuous phase of the signal. Specifically, the frequency deviation and bandwidth of the frequency scaled signal[2] in the case of sinusoidal FM modulation are given by

$$\tilde{\omega}_m = \frac{\omega_m}{R} \quad \tilde{\omega}_f = \frac{\omega_f}{R}. \tag{7}$$

Frequency upshifting serves the purpose of increasing the carrier frequency of the interpolated signal so that the CR/FD and CR/IB parameters of the signal are increased to regimes where the ESA performs well. Furthermore, the filtering operation used in the heterodyne step combined with the bandwidth compression afforded by the scaling step allows for efficient noise shaping similar to that used for quantization [6]. Specifically, the CR/FD and CR/IB parameters of $y_{\text{upsh}}(t)$ are given by

$$\begin{aligned}\left[\frac{\text{CR}}{\text{FD}}\right]_{\text{upsh}} &= \left[\frac{\text{CR}}{\text{FD}}\right]_{\text{old}} + \frac{\omega_d R}{\omega_m}\\ \left[\frac{\text{CR}}{\text{IB}}\right]_{\text{upsh}} &= \left[\frac{\text{CR}}{\text{IB}}\right]_{\text{old}} + \frac{\omega_d R}{\omega_f}.\end{aligned} \tag{8}$$

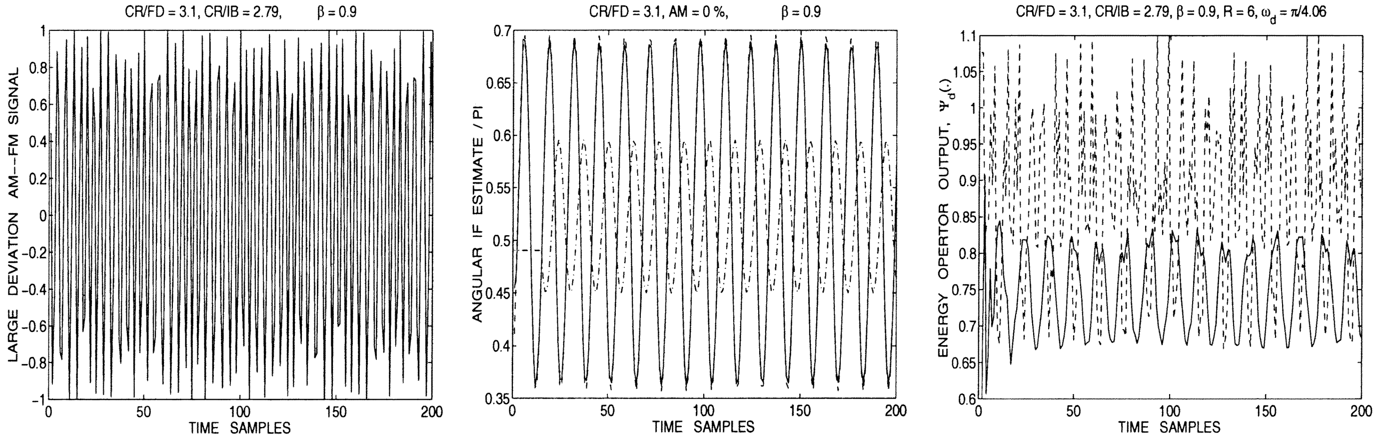
The combination of the two operations results in a signal that has a smaller frequency deviation, larger CR/FD and CR/IB ratios, larger SNR, and consequently produces a smaller error. For the discrete case, the frequency compression/expansion operations are replaced with the multirate operations of interpolation, and decimation and the discrete versions of the ESA (DESA) [4], [5] are employed. Both the decimation and interpolation operations are then implemented efficiently using a polyphase decomposition for the filters.[3] By combining this multirate framework with binomial energy smoothing, significant gains can be achieved in terms of IF demodulation error relative to the standard ESA. Although the analysis done here was for sinusoidal FM, there is no loss of generality for other types of modulation.

III. Simulation Results

Consider the sinusoidal FM example in Fig. 1, where CR/IB $= 10$ and $\beta = 10$, which falls in the wideband-FM regime where the regular ESA fails. The large deviation FM signal of the example is described in Fig. 1(a). The IF estimates of the proposed ESA-LDEV approach using the parameters $R = 1.96$, $L = 32$, and $\omega_d = \pi/2.875$ are shown in Fig. 1(b) along with estimates of the standard ESA. Note that the actual IF spans the entire range of frequencies in [0, π]. Using the DESA-1, we obtain an improvement of 21.3 dB in the normalized RMS IF demodulation error (NRMSE) relative to the standard DESA-1 algorithm. This improvement can be understood further upon application of (8), which

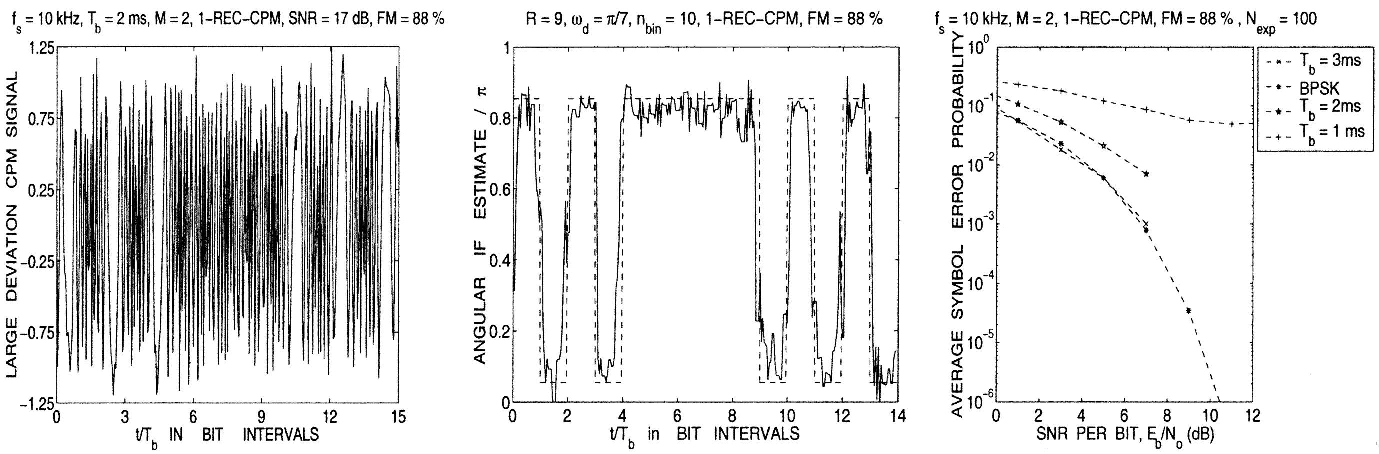
[2]The modulation index β is invariant to the above transformations.

[3]A noninteger rate change factor R can implemented via a combination of the decimation and interpolation operations with polyphase decompositions for the filters and use of the Noble identities [6].



(a) (b) (c)

Fig. 2. AM-FM demodulation for large frequency deviations. (a) FM signal with 32% FM and $\beta = 0.9$. (b) IF estimates of (solid line) the ESA-LDEV algorithm, (dashed–dotted line) the conventional ESA, and (dashed line) and actual IF. (c) Energy-operator ouput of (dashed line) the original FM signal and (solid line) the transformed signal depicting the effect of the multirate and heterodyne operations.



(a) (b) (c)

Fig. 3. Application of energy demodulation for large deviations to CPM demodulation. (a) 1-REC-CPM signal with 88% FM modulation, sampling frequency $f_s = 10$ kHz, $T_b = 2$ ms, and SNR of 17 dB. (b) IF estimates of the proposed algorithm with (solid line) five-point median smoothing and (dashed lines) the actual IF quantities. (c) Average probability of symbol error of the proposed algorithm for different symbol durations.

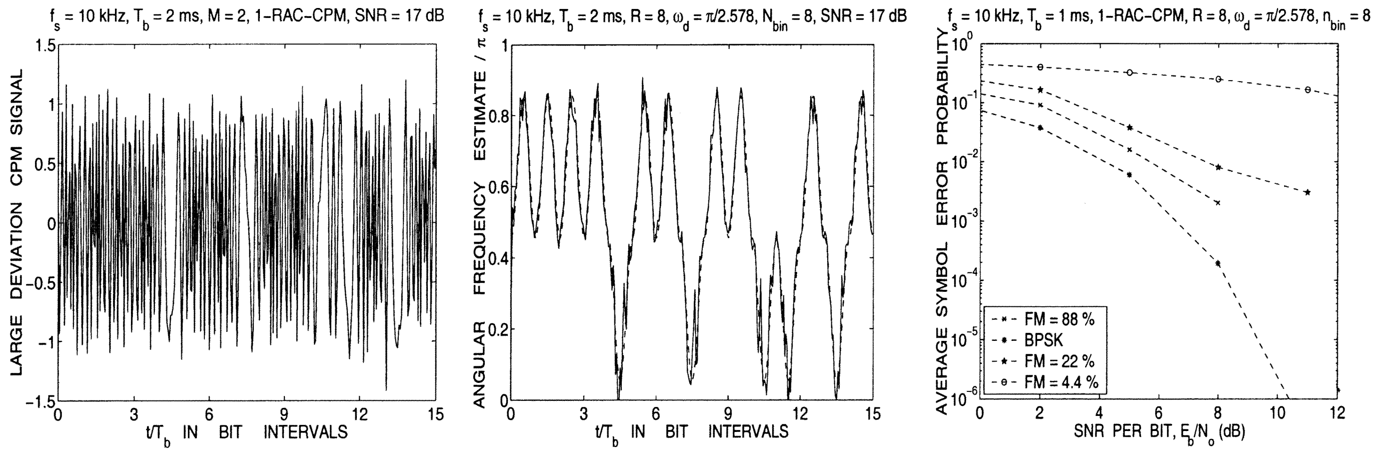
specifies a small increase in the CR/FD ratio by 1.44 but a more significant increase in the CR/IB ratio by 14.36. The improvement obtained with a noninteger rate change factor can be attributed to more accurate interpolation of the original FM signal. Fig. 1(c) describes the energy operator output of the large modulation index and the transformed signal that is greater than zero and exhibits fluctuations of longer duration compared to the energy of the original FM signal.

The improvement of the ESA-LDEV approach over the conventional ESA is further illustrated in the sinusoidal-FM example in Fig. 2, where the input signal is FM modulated with 32% FM and $\beta = 0.9$. The Carson bandwidth[4] of this signal is given by $\mathrm{BW} = 2(\omega_m + \omega_f) = 4\omega_m \approx 2\pi/3$, which indicates significant signal energy distributed over the entire spectral range. Fig. 2(b) describes the IF estimate of the proposed ESA-LDEV procedure relative to the estimates of the regular ESA for $R = 6$, $\omega_d = \pi/4.06$, and finite-impulse response filters of order $L = 34$ in the polyphase implementation of the decimator and interpolator, while a finite-impulse response bandpass filter of order $L_{\mathrm{ord}} = 420$ (Kaiser window) was used in the ESA-LDEV algorithm. The ESA-LDEV algorithm here provides 24.54-dB improvement in the NMRSE when using the DESA-1 and 33-dB improvement with DESA-2. For this example, application of (8) specifies an increase in the CR/FD parameter by approximately 9 and an increase in the CR/IB parameter by approximately 10. Fig. 2(d) describes the energy-operator output of the original FM signal and the transformed signal that exhibits fluctuations smaller in magnitude and smoother in comparison to the energy of the original FM signal.

One plausible application of the ESA-LDEV approach is to the problem of demodulating *continuous phase modulation* (CPM) signals with large frequency deviations [1], [2], [8]. For discussion purposes, we will adopt a rectangular pulse-shaping function $p(t)$ with a duration of L symbol periods (L-REC) and binary PAM symbols $a[k] \in \{-1, 1\}$. If $p(t)$ is a raised cosine pulse, then this form of CPM is referred to as (L-RAC) CPM. The IF signal in either case takes the form

$$\omega_i(t) = \omega_c + 2\pi h \sum_{k=-\infty}^{\infty} a[k] p(t - kT_b) \tag{9}$$

[4]This is defined as the difference between the frequencies where the spectral magnitude of the signal is 1% of its maximum value [3].



(a) (b) (c)

Fig. 4. 1-RAC-CPM demodulation. (a) 1-RAC-CPM signal with 88% FM modulation and SNR $=$ 17 dB. (b) Normalized IF estimates of (solid line) the proposed ESA-LDEV algorithm) with five–part median smoothing and (dashed line) the actual IF. (c) Performance of the ESA-LDEV algorithm for different amounts of FM modulation averaged over 100 experiments.

where ω_c is the carrier frequency and h is the modulation index of CPM. The phase deviation from the carrier phase is given by

$$\phi_{\rm dev}(t;\mathbf{a}) = 2\pi h \sum_{k=-\infty}^{\infty} a[k]q(t-kT_b) \tag{10}$$

where $q(t)=\int_0^t p(\tau)d\tau$ corresponds to the phase pulse-shaping function. The CPM signal is then obtained via frequency modulation. Using a pulse-shaping function of duration larger than a symbol period introduces further memory into the modulation scheme (LREC-CPM). In this letter, we will focus our attention on the case with $L=1$, i.e., (1REC-CPM). Specifically, CPM with a rectangular pulse of one symbol duration (1-REC-CPM) is equivalent to continuous phase *frequency shift keying* (CPFSK). Another form of CPM, *minimum shift keying* (MSK), is equivalent to 1-REC-CPM with $h=0.5$, while GMSK can be put into the CPM framework with a Gaussian pulse function [1].

Fig. 3 describes a CPM demodulation example with 88% FM, a sampling frequency of $f_s = 10$ kHz, $T_b = 1$ ms, where the CPM signal is described in Fig. 3(a). Fig. 3(b) describes the normalized IF estimate of the ESA-LDEV approach and the actual IF, which indicates significant wideband spectral content. Fig. 3(c) describes the performance of the ESA-LDEV approach in terms of the average symbol error probability for different symbol periods.[5] Increasing the symbol period or the sampling frequency decreases the average symbol error probability because we are using more samples per bit. Also note that the average probability of symbol error of the ESA-LDEV approach for BENRs above a threshold of 7–8 dB is zero.

Consider the example in Fig. 4, where the proposed algorithm with $R=8$, $\omega_d=\pi/2.578$ is applied to a 1-RAC-CPM signal with $f_s = 10$ kHz, $T_b = 2$ ms, and SNR $=$ 17 dB. The normalized IF estimate of the ESA-LDEV algorithm along with the actual IF is described in Fig. 4(b). The performance of the ESA-LDEV approach employing eight-time binomial energy smoothing is depicted in Fig. 4(c) for different amounts of FM modulation. A decrease in the amount of FM modulation causes a deterioration in the performance because the signal modulations are weaker.

[5]We compare the average symbol error probability to that of binary antipodal modulation in AWGN to study the effectiveness of the approach in inverting the FM modulation.

IV. Conclusion

In this letter, a generalized version of the ESA that combines frequency scaling transformations and heterodyning was described. This ESA-LDEV approach combines interpolation/decimation and heterodyne operations with simple binomial smoothing of the ESA energy signals to yield significant reduction in the normalized demodulation errors relative to the regular ESA, specifically for large frequency deviations and modulation indices. The frequency scaling operations were implemented efficiently using a polyphase representation for the interpolation/decimation operations. The efficacy of the approach was demonstrated using sinusoidal FM signals and CPM signals with large deviations and modulation indices.

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