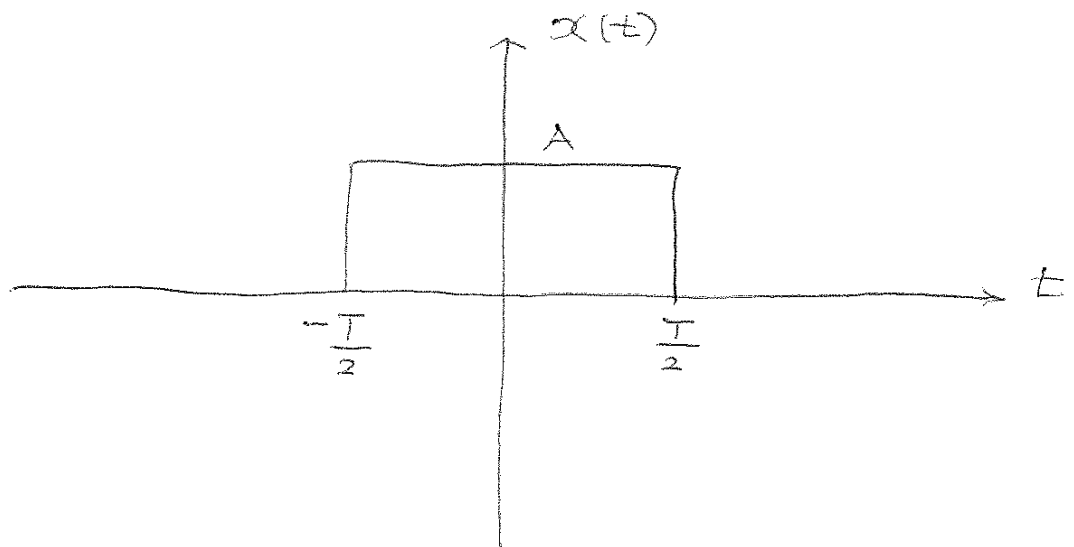


ECE - 314, FALL 2018

SIGNALS & SYSTEMS

EXAMPLE : CONTINUOUS TIME FOURIER TRANSFORM



Consider the rectangular signal

$$x(t) = r_T(t) = \begin{cases} A, & |t| < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$

The CTFT of this signal
is given by

$$X(j\omega) = A \int_{-T/2}^{T/2} \exp(-j\omega t) dt$$

Upon evaluation this integral yields

$$X(j\omega) = A \int_{-T/2}^{T/2} \frac{e^{-j\omega t}}{-j\omega} dt$$
$$= \frac{A}{-j\omega} \left\{ e^{-j\omega \frac{T}{2}} - e^{j\omega \frac{T}{2}} \right\}$$

Using Euler identities:

$$e^{-j\theta} - e^{j\theta} = -2j \sin \theta$$

$$X(j\omega) = \frac{A}{-j\omega} \cdot -2j \sin\left(\frac{\omega T}{2}\right)$$
$$= \frac{AT}{\left(\frac{\omega T}{2}\right)} \sin\left(\frac{\omega T}{2}\right)$$

Using the sinc notation:

$$X(j\omega) = AT \operatorname{Sa}\left(\frac{\omega T}{2}\right)$$

or

$$X(j\omega) = \frac{AT}{\pi} \operatorname{sinc}\left(\frac{\omega T}{2\pi}\right), \text{ where}$$

$$\text{Sa}(x) \triangleq \frac{\sin x}{x}$$

$$\text{sinc}(x) \triangleq \frac{\sin(\pi x)}{\pi x}$$

This gives us the CTFT pair

$$r_T(t) \xrightleftharpoons[\mathcal{F}^{-1}]{\mathcal{F}} \text{AT Sa}\left(\frac{\omega T}{2}\right)$$

