ECE-314, Fall 2018
Signals & Systems

Example: Discrete-Time Fourier Transform

Consider the discrete-time sequence

$$x[n] = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$$

By definition the DTFT of this sequence is given by:

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$$

$$= \sum_{n=0}^{N-1} (e^{-j\Omega})^{n}$$

Using the Geometric Progression sum:

$$X(e^{j\Omega}) = \frac{1-a^N}{1-a} = \frac{1-e^{-j\Omega N}}{1-e^{-j\Omega}}$$

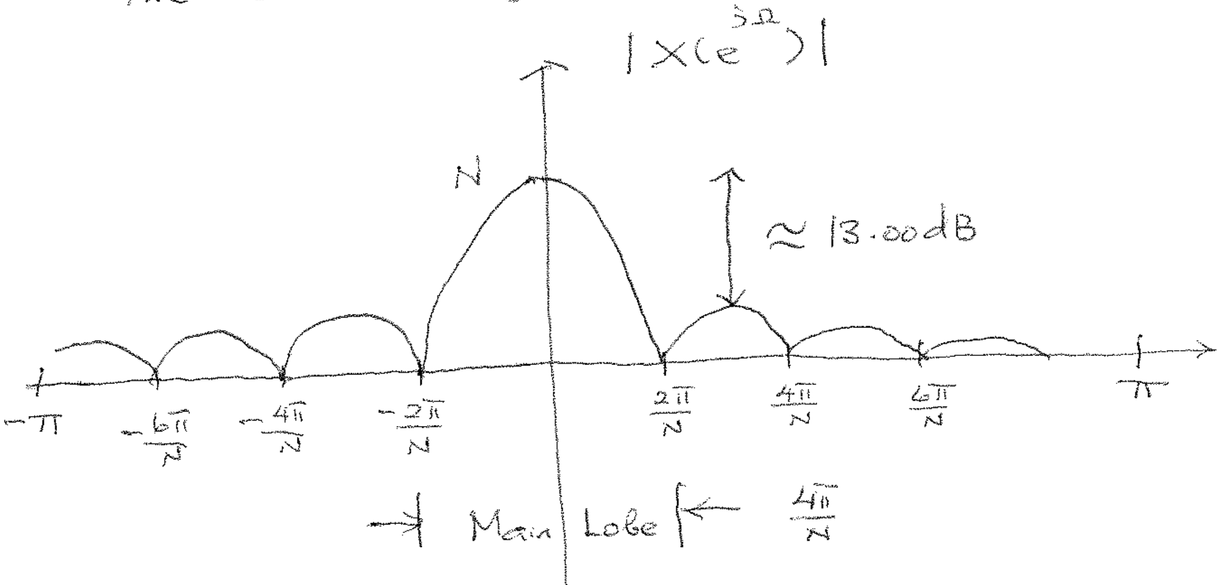
This expression can be rewritten as:

$$X(e^{j\Omega}) = \frac{e^{-j\Omega\frac{N}{2}}\left\{e^{j\Omega\frac{N}{2}} - e^{-j\Omega\frac{N}{2}}\right\}}{e^{-j\frac{\Omega}{2}}\left\{e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}\right\}}$$

$$X(e^{j\Omega}) = e^{-j\Omega\left(\frac{N-1}{2}\right)} \frac{\cancel{2j}\sin\left(\frac{\Omega N}{2}\right)}{\cancel{2j}\sin\left(\frac{\Omega}{2}\right)}$$

$$X(e^{j\Omega}) = \left[\frac{\sin\left(\frac{\Omega N}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)}\right] e^{-j\Omega\left(\frac{N-1}{2}\right)}$$

This is called the asinc function or the Dirichlet function



We therefore obtain the DTFT pair:

$$x[n] = u[n] - u[n-N] \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} e^{-j\Omega\left(\frac{N-1}{2}\right)} \frac{\sin\left(\frac{\Omega N}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)}$$