

ECE - 314 , FALL 2018

SIGNALS & SYSTEMS

APPLICATIONS OF CONVOLUTION:

CROSS - CORRELATION:

This quantity is defined as:

$$\begin{aligned} \Gamma_{xy}[k] &= x[k] * y^*[-k] \\ &= \sum_{m=-\infty}^{\infty} x[m] y^*[m-k] \end{aligned}$$

$$\begin{aligned} R_{xy}[\tau] &= x(\tau) * y^*(-\tau) \\ &= \int_{-\infty}^{\infty} x(\alpha) y^*(\alpha - \tau) d\alpha \end{aligned}$$

This quantity measures the similarity between signals $x(t)$ and $y(t)$

AUTO - CORRELATION:

$$\begin{aligned} \Gamma_{xx}[k] &= x[k] * x^*[-k] \\ &= \sum_{m=-\infty}^{\infty} x[m] x^*[m-k] \end{aligned}$$

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(\tau) x^*(\tau - \tau) d\tau$$

Two waveforms are said to be uncorrelated iff:

$$(i) R_{xy}(\tau) = 0, \quad \forall \tau \in \mathbb{R}$$

$$(ii) \Gamma_{xy}[k] = 0, \quad \forall k \in \mathbb{Z}^{\pm}$$

Two waveforms are orthogonal:

$$\begin{aligned} \langle x(t), y(t) \rangle &= R_{xy}(0) = \int_{-\infty}^{\infty} x(\tau) y^*(\tau) d\tau \\ &= 0 \end{aligned}$$

$$\langle x[k], y[k] \rangle = \Gamma_{xy}[0]$$

$$= \sum_{m=-\infty}^{\infty} x[m] y^*[m] = 0$$

This measures the similarity between two waveforms. Orthogonal signals have no similarity

The quantities, $R_{xx}(0)$, $r_{xx}[0]$ correspond to the energy of the waveform:

$$R_{xx}(0) = \int_{-\infty}^{\infty} |x(t)|^2 dt = E_x$$

$$r_{xx}[0] = \sum_{m=-\infty}^{\infty} |x[m]|^2 = E_x$$

For this reason: $R_{xx}(\tau)$, $r_{xx}[k]$ are called energy auto-correlations