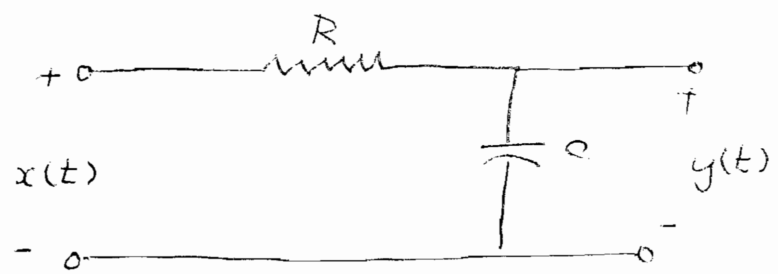
# ECE-314, Fall 2008
# Signals and Systems

## Example: Stability & Causality

Consider the R-C circuit discussed before




$$x(t) = i(t)R + y(t)$$

$$x(t) = RC\frac{dy}{dt} + y(t)$$

Let us look at the response $h(t)$ to a Dirac impulse at $t=0$, i.e., $h(t) = L(\delta(t))$:

$$RC\frac{dh}{dt} + h(t) = \delta(t) \qquad (1)$$

For $t > 0_+$

$$RC\frac{dh}{dt} + h(t) = 0 \qquad (2)$$

The solution to this DE can be obtained by substituting $h(t) = e^{-\lambda t}$

$$RC\left(-\lambda e^{-\lambda t}\right) + e^{-\lambda t} = 0$$

$$\Longrightarrow \quad -\lambda RC + 1 = 0$$

$$\text{or} \quad \lambda = 1/RC$$

The solution to the unforced system is of the form

$$h(t) = c_0 e^{-t/RC}$$

Heuristically speaking for the forced system the solution needs to be of the form:

$$h(t) = c_0 e^{-t/RC} u(t) \qquad \left( \frac{du}{dt} = \delta(t) \right)$$

whence

$$h'(t) = -\frac{c_0}{RC} e^{-t/RC} u(t) + c_0 e^{-t/RC} \delta(t)$$

$$\text{or} \qquad h'(t) = -\frac{c_0}{RC} e^{-t/RC} u(t) + c_0 \delta(t)$$

$$RC\, h'(t) = -c_0 \cancel{e^{-t/RC}} u(t) + RC\, c_0\, \delta(t)$$

$$+ \qquad h(t) = c_0 \cancel{e^{-t/RC}} u(t)$$

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$$RC\, h'(t) + h(t) = RC\, c_0\, \delta(t) = \delta(t) \quad \text{by (.)}$$

$$\Longrightarrow \quad c_0 = \frac{1}{RC}$$

- The impulse response of the R-C voltage divider circuit is:

$$h(t) = \frac{1}{RC} e^{-t/RC} u(t)$$

- Since $h(t) = 0$, $t < 0$; the R-C voltage divider circuit is causal

- Also $$\int_0^\infty \frac{1}{RC} e^{-t/RC} \, dt = 1 < \infty,$$

which means the R-C circuit is BIBO stable.

- This result is intuitive in that voltage division cannot result in a unbounded output and the maximum allowable gain out of voltage division is unity.