ECE-314, FALL 2008

Signals & Systems

Example: DTFT

Consider the LTI system characterized by the difference equation:

$$y[n] = a\,y[n-1] + x[n], \quad \text{where}$$
$$y[n] = 0, \; n \leq -1, \quad a \in \mathbb{R}^1 > 0$$

Taking the DTFT on both sides:

$$Y(e^{j\Omega}) = a\,e^{-j\Omega}Y(e^{j\Omega}) + X(e^{j\Omega})$$

or

$$Y(e^{j\Omega})(1 - a e^{-j\Omega}) = X(e^{j\Omega})$$

or

$$\frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \frac{1}{1 - a e^{-j\Omega}}, \quad |\Omega| < \pi$$

or

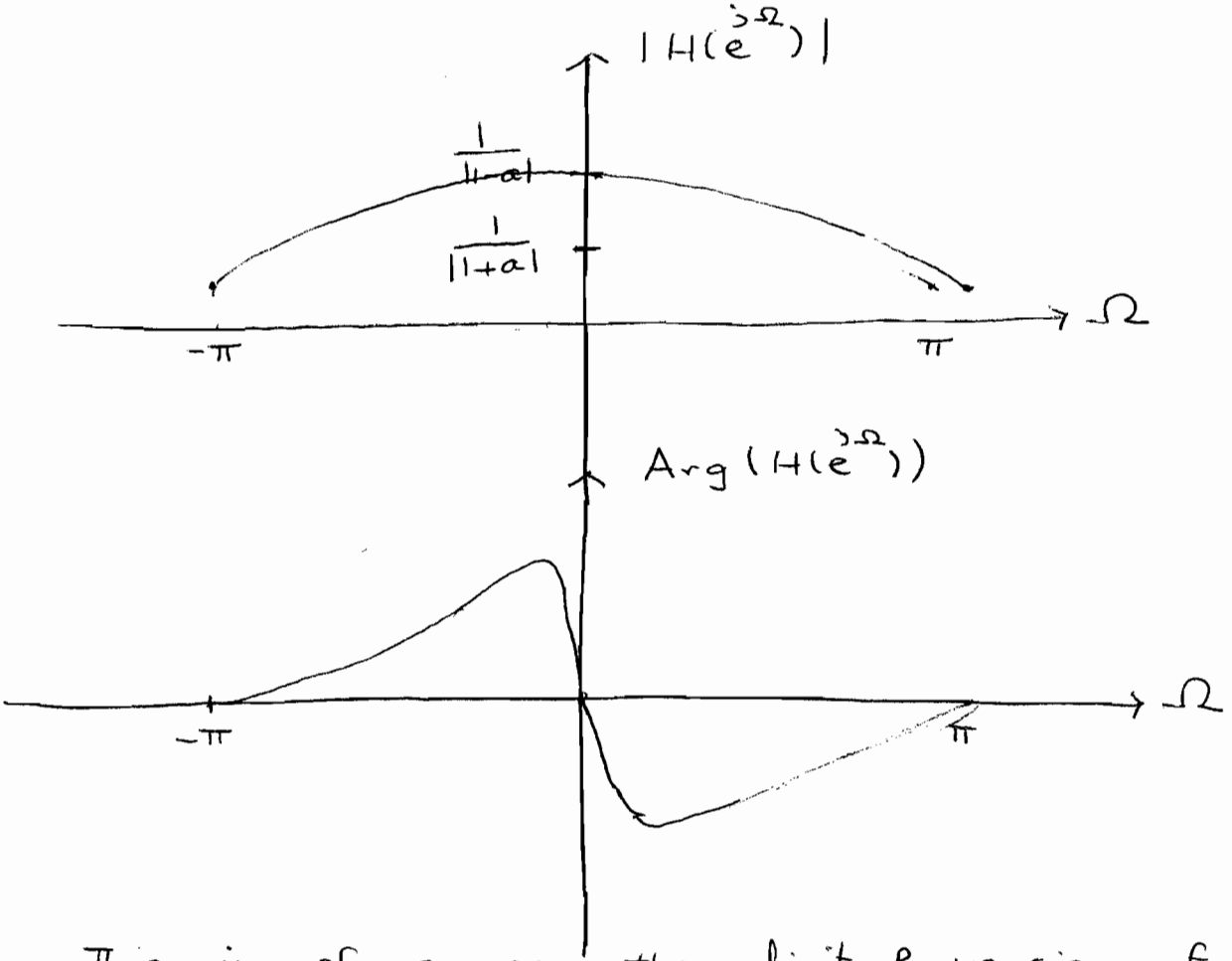
$$H(e^{j\Omega}) = \frac{1}{1 - a e^{-j\Omega}}, \quad |\Omega| < \pi$$

Magnitude & Phase Response

$$H(e^{j\Omega}) = \frac{1}{1 - a\cos\Omega + j\,a\sin\Omega}$$

$$|H(e^{j\Omega})| = \frac{1}{\sqrt{1 + a^2 - 2a\cos\Omega}}$$

$$\text{Arg}\,(H(e^{j\Omega})) = -\tan^{-1}\left(\frac{a\sin\Omega}{1 - a\cos\Omega}\right)$$



This is of course, the digital version of a LPF

The corresponding impulse response:

$$h[n] = \text{IDTFT}\left\{\frac{1}{1-ae^{-j\Omega}}\right\}, \quad a<1$$

$$= a^n u[n]$$

This ofcourse is a stable & causal system for $|a|<1$