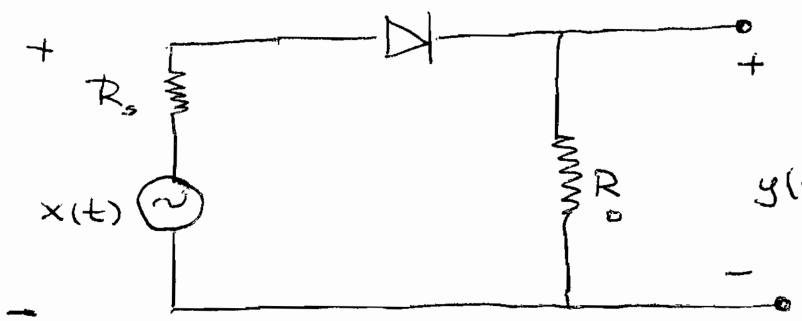
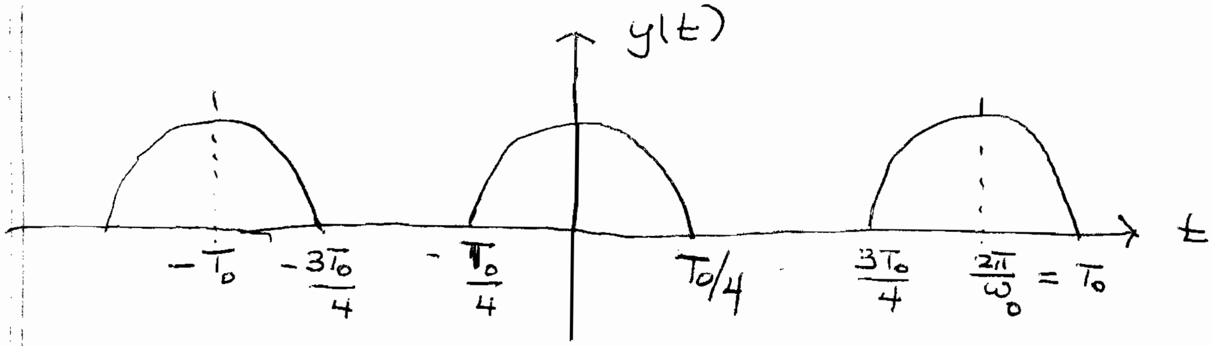
ECE-314, Fall 2008
Signals and Systems

Example: Half Wave Rectifier



$$y(t) = \begin{cases} x(t), & x(t) \geq 0 \quad \text{(Forward Bias)} \\ 0, & \text{otherwise (Reverse Bias)} \end{cases}$$

Specifically if $x(t) = \cos(\omega_0 t)$ then
$y(t) = |\cos(\omega_0 t)|$



For the input $x(t)$:

$$x(t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$$

$$\left.\begin{array}{l} C_1 = \frac{1}{2} \\ C_{-1} = \frac{1}{2} \end{array}\right\} \quad C_k = 0 , \; k \neq \pm 1$$

For the output :

$$d_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \cos(\omega_0 t) \, e^{-jk\omega_0 t} \, dt$$

$$d_k = \frac{1}{T_0} \int_{-T_0/4}^{T_0/4} \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right) e^{-jk\omega_0 t} \, dt$$

$$d_k = \frac{1}{2T_0} \int_{-T_0/4}^{T_0/4} e^{j\omega_0 t(1-k)} \, dt$$

$$+ \frac{1}{2T_0} \int_{-T_0/4}^{T_0/4} e^{-j\omega_0 t(1+k)} \, dt$$

$$d_k = \frac{1}{2T_0} \left. \frac{e^{j\omega_0 t(1-k)}}{j\omega_0(1-k)} \right|_{-T_0/4}^{T_0/4}$$

$$+ \frac{1}{2T_0} \left. \frac{e^{-j\omega_0 t(1+k)}}{-j\omega_0(1+k)} \right|_{-T_0/4}^{T_0/4}$$

$$d_k = -\frac{1}{\cancel{2j}\, 2\pi(1-k)} \cancel{2j} \sin\left(\frac{\pi}{2}(1-k)\right)$$

$$- \frac{1}{\cancel{2j}\, 2\pi(1+k)} - \cancel{2j} \sin\left(\frac{\pi}{2}(1+k)\right)$$

$$d_k = \frac{1}{2\pi(k+1)} \sin\left(\frac{\pi}{2} + \frac{\pi}{2}k\right)$$

$$- \frac{1}{2\pi(1-k)} \sin\left(\frac{\pi}{2} - \frac{\pi}{2}k\right)$$

$$d_k = \frac{1}{2\pi(k+1)} \cos\left(k\frac{\pi}{2}\right) + \frac{1}{2\pi(k-1)} \cos\left(k\frac{\pi}{2}\right)$$

$$d_k = \frac{1}{2\pi} \frac{2k}{k^2-1} \cos\left(\frac{k\pi}{2}\right)$$

$$x(t) = \sum_{k=-\infty}^{\infty} d_k \exp\{jk\omega_0 t\}$$

Observations:

- Input has only 2 harmonics, output has an infinite number of harmonics
- Odd harmonics at $k = \pm 1$ present in input absent in output
- No odd harmonics in output