ECE - 314, Fall 2008
Signals and Systems

Example: Classification

Consider the input - output map:

$y(t) = t\,x(t)$, where $x(t)$ is the input and $y(t)$ is the output

(a) Linearity: Look at $L(c_1 x_1(t) + c_2 x_2(t))$ where $c_1 \neq 0$, $c_2 \neq 0$

$$\begin{aligned} L(c_1 x_1(t) + c_2 x_2(t)) &= t\,(c_1 x_1(t) + c_2 x_2(t)) \\ &= c_1\, t\, x_1(t) + c_2\, t\, x_2(t) \\ &= c_1\, L(x_1(t)) + c_2\, L(x_2(t)) \end{aligned}$$

$\Rightarrow$ POS holds and $L(\cdot)$ is linear

(b) Time - invariance:

$$L(x(t-t_0)),\ t_0 \in \mathbb{R} = t\,x(t-t_0)$$
$$y(t-t_0),\ t_0 \in \mathbb{R}^1 = (t-t_0)\,x(t-t_0)$$

Since $y(t-t_0) \neq L(x(t-t_0))$, system is not a TI system

(c) Stability

Suppose $|x(t)| < B_1$, $t \in \mathbb{R}$
$|y(t)| < t B_1$

However, we cannot write $|y(t)| < B_2$ for $t \in \mathbb{R}^1$, since t cannot be bounded for any $t \in \mathbb{R}^1$

$\Rightarrow$ $L(.)$ is not a BIBO map.

(d) Causality: This system is causal because for any $t_0 \in \mathbb{R}^1$, knowledge of t_0, $x(t_0)$ is sufficient to determine $y(t_0)$:

$$y(t_0) = t_0\, x(t_0), \quad t_0 \in \mathbb{R}^1$$