Final Exam, Fall 2018

Signals and Systems, ECE-314
University of New Mexico
Instructor: Balu Santhanam
Date Assigned: 12/12/2018, 8:00 AM, Wednesday
Due Back : 12/13/2018, Thursday, 8:00 AM.

Instructions

1. Pick any 4 out of the 6 questions assigned. Pick only 4. The first four will be selected and the others discarded.
2. Write clearly and legibly. Chicken scratch is hazardous to both the student and the professor.
3. Provide steps to obtain partial credit
4. It is assumed that you are aware of the UNM academic honesty policy. Needless to say copying or collaborative work of any kind will be dealt with seriously.
5. Exam is open book, open notes, you may use MATLAB to check answers.

Problem # 1.0

The impulse response $h(t)$ and the input to a continuous time LTI system are given by:

$$h(t) = \begin{cases} 1 & t \in [0, 3] \\ 0 & \text{otherwise} \end{cases}$$

and

$$x(t) = \begin{cases} 1 & t \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

1. Compute the output of this system by evaluating the convolution integral via the "flip and slide" method. Clearly identify the various intervals of the output over which you are calculating the integral.

2. If instead the input to the system is given by:

$$\tilde{x}(t) = \begin{cases} -1 & t \in [0, 1] \\ 2 & t \in [1, 2], \end{cases}$$

   evaluate the corresponding output $\tilde{y}(t)$ without computing the convolution integral again.

Problem # 2.0

A discrete-time LTI system has a system function of the form:

$$H(z) = \frac{1}{(1 - 3z^{-1})(1 - 0.5z^{-1})^2}.$$

For this system:

1. draw the pole-zero plot of $H(z)$
2. determine the possibilities for the region of convergence of $H(z)$ and determine the ROC possibility that produces: (1) a BIBO stable system and (b) a causal system.
3. Compute the impulse response $h[n]$ for the BIBO stable system from the previous part.
4. Does this system have a stable and causal inverse system? Justify your answer.

Problem # 3.0

Compute the Fourier transform of the signals given below. In each case specify the transform pairs that you are using and the properties of the Fourier transform that you are using. For continuous signals, the Fourier transform refers to the CTFT and for discrete signals it refers to the DTFT.

1. $x(t) = t^n e^{-at} \cos(\omega_o t) u(t)$, with $a > 0$.
2. $x(t) = e^{-t^2/2} \cos(\omega_o t)$.
3. $x[n] = a^{|n|} \cos(\Omega_o n)$, with $|a| < 1$.
4. $x[n] = \begin{cases} \sin(\Omega_o n) & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$

Problem # 4.0

A continuous-time, causal and stable LTI system has a transfer function of the form:

$$H(s) = \frac{s-1}{(s+1)^2}.$$

1. Draw the pole zero-plot for $H(s)$ and determine the appropriate region of convergence (ROC).

2. Determine the frequency response $H(j\omega)$ of this system. What is the corresponding input-output differential equation satisfied by this system?

3. Determine the impulse response $h(t)$ of this system.

4. If the input to the system is $x(t) = \exp(-2t)\, u(t)$ compute the corresponding output $y(t)$.

Problem # 5.0

The input to both a half-wave rectifier and a full-wave rectifier circuit is a sinusoidal signal $x(t) = \cos(\omega_c t + \theta)$. The output of the half-wave rectifier is given by:

$$y(t) = L_1(x(t)) = \begin{cases} x(t) & x(t) \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

while the output of the full-wave rectifier circuit is given by:

$$y(t) = L_2(x(t)) = |x(t)|.$$

1. Determine if the two input-output mappings L_1 and L_2 are linear.

2. Develop a complex Fourier series expansion for the output of both circuits for this input signal by computing the Fourier series coefficients of the expansion.

3. Comment on the difference between the outputs in both cases.

Problem # 6.0

A continuous-time signal $x_c(t)$ has a finite duration $|t| < t_M$. The spectrum of this time-limited waveform is $X_c(j\omega)$. Our goal is to sample this spectrum in frequency at a discrete set of points $\omega = k\Omega_o$ using impulse-sampling:

$$\tilde{X}_s(j\omega) = X_c(j\omega)P(j\omega) = X_c(j\omega) \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_o)$$

1. Compute the inverse Fourier transform of the frequency-domain impulse-sampling pulse:
$$P(j\omega) = \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_o).$$

2. Use convolution theorem to develop a relationship between the waveforms $\tilde{x}_s(t)$ and $x_c(t)$ analogous to the alias-sum formula for time-domain sapling.

3. How should the time-gate parameter t_M be chosen so that there is no loss of information.