ECE314: Signals and Systems
Knowledge Probe - 2007 Spring

Instructions:

1. The Purpose of this Knowledge Probe is to get a better assessment of your prerequisites. Please **answer all questions** to the best of your capability.

2. For any question you **does not completely solve**, please indicate the best explanation from the list below:

   A. I have no idea how to solve this problem.

   B. I think I could solve this problem, if I were given more time.

   C. I learnt how to solve problems like this in course titled **"xxxxx"** that I took in Fall/Spring/Summer semester of year **"xxxxx"**, but right now I have forgotten the techniques.

3. Whenever possible, please indicate in which class you learnt to solve each of the questions and when you took that class

4. Strictly no calculators are allowed.

Q1. Write the following complex number in the polar form (Note: $j = \sqrt{-1}$):

$$3 + j2 + 2\exp\left(j\frac{\pi}{2}\right)$$

Q2. Solve the following differential equation for $t \geq -1$. The initial condition is given as $x(-1) = e$ (Note: e is the base of the natural logarithm).

$$\frac{d}{dt}x(t) \;=\; 2\,t\,x(t)$$

Q3. Solve the following integrals:

(a).

$$\int_{-\infty}^{\infty} \exp\left(-2|t| - 1\right) dt$$

(b).

$$\int_{-\infty}^{a} \frac{1}{1 + \pi^2 x^2}\,dx$$

Hints: $\frac{d}{dx}\tan^{-1}(x) \;=\; \frac{1}{1+x^2}$, and $\tan^{-1}(-\infty) \;=\; -\frac{\pi}{2}$.

Q4. Suppose that,

$$\int_{-\infty}^{\infty} x(t) \exp\left(-j\pi t\right) dt \; = \; A.$$

Solve the following integral in terms of A.

$$\int_{-\infty}^{\infty} x(t+1) \exp\left(-j\pi t\right) dt$$

Q5. Assuming zero initial conditions, find $y(t)$ from the following 2nd order differential equation using Laplace Transform method:
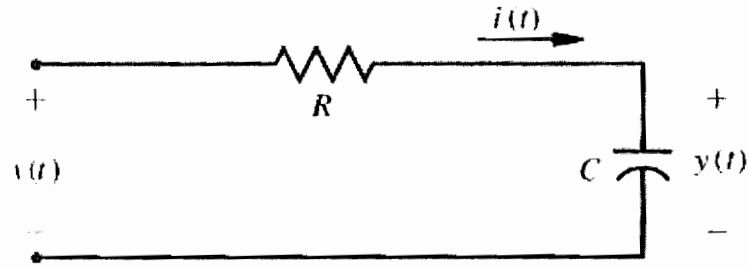
$$\frac{d^2}{dt^2} y(t) + 3\frac{d}{dt} y(t) + 2y(t) \; = \; 4 \, e^{-3t} u(t)$$

where $u(t)$ is the unit step function.

Hint: $\mathcal{L}\left\{e^{-at} u(t)\right\} = \frac{1}{s+a}$, where $\mathcal{L}\{.\}$ denotes the Laplace Transform.

Q6. If $\sin(30^o) = \frac{1}{2}$ and $\sin(45^o) = \frac{1}{\sqrt{2}}$, can you find $\sin(75^o)$ using trigonometric identities?

Q7. Suppose that the input and output voltages in the following circuit are $x(t)$ and $y(t)$, respectively. Write down the system equation.

(Note: You do not need to solve the system equation)

Hint: System equation is a relationship between the input and output.