ECE-314, Fall 2008
Signals and Systems
Example: LTI system

Consider the system map $L(\cdot)$ given by the expression:

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = L(x(t))$$

Suppose $y_1(t) = L(x_1(t))$
$y_2(t) = L(x_2(t))$

Let $x_3(t) = c_1 x_1(t) + c_2 x_2(t)$

$$L(x_3(t)) = \int_{-\infty}^{\infty} h(\tau)\left(c_1 x_1(t-\tau) + c_2 x_2(t-\tau)\right) d\tau$$

$$L(x_3(t)) = \int_{-\infty}^{\infty} h(\tau)\, c_1 x_1(t-\tau)\, d\tau + \int_{-\infty}^{\infty} h(\tau)\, c_2 x_2(t-\tau)\, d\tau$$

$$L(x_3(t)) = c_1 \int_{-\infty}^{\infty} h(\tau)\, x_1(t-\tau)\, d\tau + c_2 \int_{-\infty}^{\infty} h(\tau)\, x_2(t-\tau)\, d\tau$$

$$= c_1 y_1(t) + c_2 y_2(t)$$

$\Rightarrow$ The mapping $L(\cdot)$ is a linear map

Now look at $L(x(t-t_0))$, $t_0 \in \mathbb{R}$

$$L(x(t-t_0)) = \int_{-\infty}^{\infty} h(\tau)\, x(t-t_0-\tau)\, d\tau$$

$$y(t-t_0) = \int_{-\infty}^{\infty} h(\tau)\, x(t-t_0-\tau)\, d\tau$$

Since $y(t-t_0) = L(x(t-t_0))$, the system map $L(\cdot)$ is a TI system

$\Rightarrow$ $L(\cdot)$ is a LTI system

$\Longrightarrow$ The system whose input-output map is the convolution Integral is a LTI system.