Midterm Exam, Fall 2018

Signals and Systems, ECE-314
University of New Mexico
Instructor: Balu Santhanam
Date Assigned: 10/22/2018, 8:00 AM, Monday
Due Back : 10/23/2018, 8:00 AM, Tuesday

Instructions

1. Pick any 4 out of the 6 questions assigned.
2. Write clearly and legibly. Chicken scratch is hazardous to both the student and the professor.
3. Provide steps to obtain partial credit
4. It is assumed that you are aware of the UNM academic honesty policy. Needless to say copying or collaborative work of any kind will be dealt with seriously.
5. Exam is open book, open notes, you may use MATLAB to check answers.

Problem # 1.0

For each of the systems listed below, determine if the corresponding input-output system map is: (a) linear, (b) time-varying, (c) causal, and (d) BIBO stable:

1. $y(t) = \frac{1}{1+\exp(-x(t))}$.
2. $y(t) = x(-t)\cos(\omega_o t)$
3. $y[n] = (-1)^n x[n]$
4. $y(t) = \frac{d}{dt}(x(2t))$
5. $y[n] = \exp(j\Omega_o n)\, x[n]$

In each case, justify your answer properly. This means if you claim the positive, provide a proof and if you claim the negative provide a counter-example.

Problem # 2.0

A LTI system who impulse response $h(t)$ is of the form:

$$h(t) = \begin{cases} 1 & t \in [0, T] \\ 0 & \text{o.w.} \end{cases}$$

is excited by an input signal $x(t)$ of the form:

$$x(t) = \begin{cases} \exp(-2t) & t \in [0, 1] \\ 0 & \text{o.w.} \end{cases}$$

For this environment:

1. Compute the output $y(t)$ of this system clearly indicating the various regions of interest in time.
2. Based on the information provided: (a) Is this system causal ? (b) Is this system BIBO stable? Justify your answers properly.

Problem # 3.0

In class, we have looked at a specific periodic signal called the *Dirac comb* that can be expressed as:

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s).$$

This signal is used to model the ideal *analog to digital conversion* (ADC) process. However for practical applications this "sampling signal" is replaced with a realistic signal of the form:

$$g(t) = \sum_{k=-\infty}^{\infty} \operatorname{rect}\left(\frac{t - kT_s}{\tau}\right),$$

which contains rectangular pulses with a small duty-cycle $\eta = \frac{\tau}{T_s}$. For this signal:

1. compute the Fourier transform $P(j\omega)$ of the Dirac comb signal $p(t)$.
2. compute the complex Fourier series coefficients $c[k]$ of the realistic sampling signal $g(t)$
3. compute the associated Fourier transform $G(j\omega)$ using the Fourier coefficients $c[k]$ calculated in the previous part and compare with result obtained in the first part.

Problem # 4.0

Consider a parallel current divider L-R-C circuit, where the input current source is $x(t)$ and the output current through the resistor is $y(t) = i_R(t)$. For this circuit system:

1. Determine the input-output differential equation, ie., the input-output map.

2. Compute the frequency response of this circuit system:

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{I_R(j\omega)}{I(j\omega)}$$

3. If the input to the system is $i(t) = \cos(\omega_o t)$ then compute the output $i_R(t)$ using the results of the previous part.

Problem # 5.0

A continuous-time LTI system has a impulse response of the form:

$$h(t) = \exp(-2t)\, u(t)$$

The input to this system is given by:

$$x(t) = \exp(-3t)\, u(t).$$

For this system:

1. compute the output of the system by direct evaluation of the convolution integral using the flip and slide method, clearly indicating various intervals in time.

2. compute the frequency response $H(j\omega)$ of this system, i.e. compute:

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) \exp(-j\omega t)\, dt.$$

3. Using the frequency response $H(j\omega)$ obtained in the previous part, determine the input-output different equation associated with this system. You can make use of the Fourier transform pair:

$$\mathcal{F}\left(\frac{d}{dt}(x(t))\right) = j\omega X(j\omega).$$

4. compute the output of the system using Fourier transforms and the results of the convolution theorem, i.e.,

$$y(t) = \mathcal{F}^{-1}\left(H(j\omega)X(j\omega)\right)$$

   You are allowed to use the Fourier transform table from your text book.

Problem # 6.0

Three discrete-time periodic sequences $x_1[n]$ and $x_2[n]$ are given by:

$$x_1[n] = \sum_{k=-\infty}^{\infty} \delta[n - kL]$$

and

$$x_2[n] = \sum_{k=-\infty}^{\infty} g[n - kL],$$

where $g[n]$ is the finite duration sequence:

$$g[n] = \begin{cases} 1 & 0 \leq n \leq M - 1 \\ 0 & \text{o.w.} \end{cases}$$

with $M < L$. The third periodic sequence $x_3[n]$ is given by:

$$x_3[n] = \sum_{k=-\infty}^{\infty} r[n - kM],$$

where $r[n] = (-1)^n g[n]$. For these three sequences: determine the discrete-time Fourier series coefficients $c_1[k]$ and $c_2[k]$ and $c_3[k]$ associated with them.