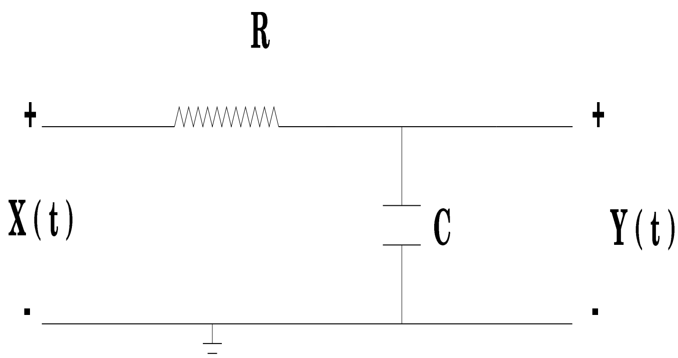

Figure 1: R–C Voltage divider circuit: $x(t)$ is the input to the system and $y(t)$ is the corresponding output. R and C are assumed constant.

R–C Voltage Divider Circuit

Consider the R–C voltage divider circuit in Fig. 1. Let us excite this system with an input voltage signal $x(t)$. Let $y(t)$ be the corresponding output voltage signal. Writing the *Kirchoff voltage law* (KVL) for the loop yields:

$$x(t) = y(t) + RC\frac{dy}{dt}. \tag{1}$$

The non-homogeneous constant coefficient differential equation describes the input–output map associated with the R–C circuit. Let $y_1(t)$ and $y_2(t)$ be the responses of this R–C circuit to the inputs $x_1(t)$ and $x_2(t)$, i.e.,

$$x_1(t) = y_1(t) + RC\frac{dy_1}{dt} \tag{2}$$

$$x_2(t) = y_2(t) + RC\frac{dy_2}{dt} \tag{3}$$

Multiplying Eq. (2) by c_1, Eq. (3) by c_2 and adding the equations together we have

$$c_1x_1(t) + c_2x_2(t) = c_1y_1(t) + c_2y_2(t) + RC\frac{d}{dt}\left(c_1y_1(t) + c_2y_2(t)\right).$$

Relabeling $x_3(t) = c_1x_1(t) + c_2x_2(t)$ and $y_3(t) = c_1y_1(t) + c_2y_2(t)$ we have

$$x_3(t) = y_3(t) + RC\frac{dy_3}{dt}. \tag{4}$$

This implies that $y_3(t)$ is the output of the circuit when the input is $x_3(t)$, i.e.,

$$c_1y_1(t) + c_2y_2(t) = \mathbf{L}(c_1x_1(t) + c_2x_2(t)).$$

This implies that the system underlying the R–C voltage divider circuit is a linear system because the *principle of superposition* (POS) holds. This is not a surprising result because the underlying differential equation in Eq. (1) is linear.

Upon multiplying the input–output differential equation in Eq. (1) with the integrating factor $\exp\left(-\frac{t}{RC}\right)$ and solving the resultant differential equation we obtain the solution

$$y(t) \;=\; \underbrace{\exp\left(-\frac{t-t_o}{RC}\right)y(t_o)}_{\text{response to initial conditions}} + \underbrace{\frac{1}{RC}\int_{t_o}^{t} x(\tau)\exp\left(-\frac{t-\tau}{RC}\right)d\tau}_{\text{response to input}}\;,\; t > t_o. \tag{5}$$

If the system is intially at rest, i.e, $y(t_o) = 0$ and $x(t) = 0, t < t_o$ then

$$y(t) \;=\; \frac{1}{RC}\int_{t_o}^{t} x(\tau)\exp\left(-\frac{t-\tau}{RC}\right)d\tau = \int_{t_o}^{t} x(\tau)\underbrace{\frac{1}{RC}\exp\left(-\frac{t-\tau}{RC}\right)}_{h(t-\tau)}d\tau.$$

This is in the form of a convolution integral where the impulse response $h_{RC}(t)$ is given by

$$h_{RC}(t) \;=\; \frac{1}{RC}\exp\left(-\frac{t}{RC}\right). \tag{6}$$

This equation implies that system underlying the R–C voltage divider circuit is a LTI system provided that the system is initially at rest.

If in addition we impose the restriction of causality on the system, i.e., present output does not depend on future output we have

$$h_{RC}(t) \;=\; \mathbf{L}(\delta(t)) \;=\; \frac{1}{RC}\exp\left(-\frac{t}{RC}\right)u(t). \tag{7}$$