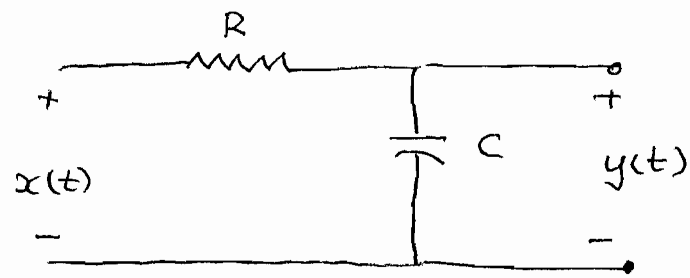
ECE-314, FALL 2008

SIGNALS & SYSTEMS

On the ideal LPF:

In class we have looked at the R-C voltage divider circuit as a LPF:



The corresponding frequency response:

$$H(j\omega) = \frac{a}{a+j\omega}, \quad \omega \in \mathbb{R}, \quad a = 1/RC$$

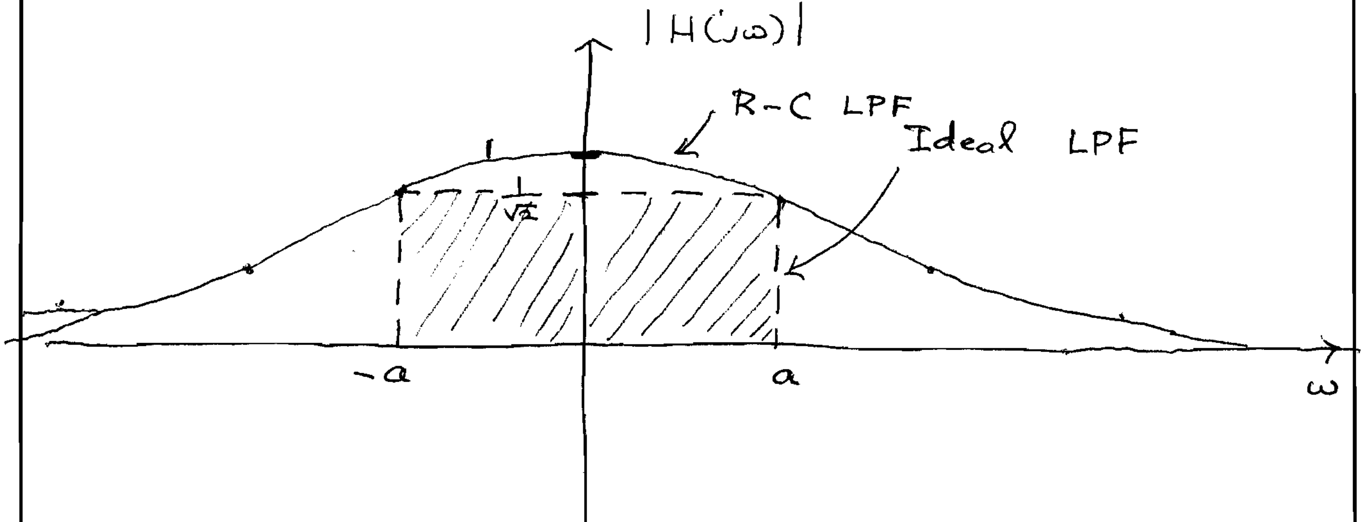
$$|H(j\omega)| = \frac{a}{\sqrt{a^2+\omega^2}}, \quad \omega_{3dB} = \pm a$$

$$\arg(H(j\omega)) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

However, this is not the response of the ideal LPF that rejects signals completely outside of $\omega \in [-a, a]$

Suppose we consider a LPF with frequency response:

$$H_{LP}(j\omega) = \begin{cases} 1, & |\omega| < a \\ 0, & \text{otherwise} \end{cases}$$



The corresponding impulse responses:

$$h_{RC}(t) = \frac{1}{RC}\, e^{-t/RC}\, u(t)$$

$$h_{LP}(t) = \frac{1}{2\pi}\int_{-a}^{a} e^{j\omega t}\, d\omega = \frac{1}{2\pi}\left.\frac{e^{j\omega t}}{jt}\right|_{-a}^{a}$$

$$h_{LP}(t) = \frac{a}{\pi}\,\frac{\sin(at)}{at}, \quad t \in \mathbb{R}$$

Note first the sharp response of the ideal LPF versus the slow decay of the R-C lowpass filter.

- The second but more serious implication is in regards to causality :

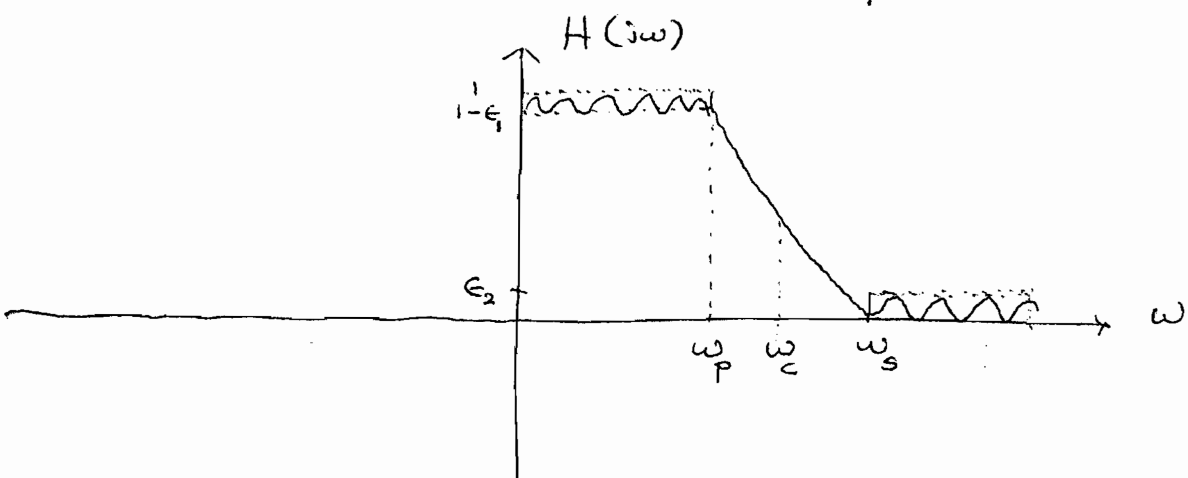
$$h_{RC}(t) = \frac{1}{RC} e^{-t/RC} u(t) = 0, \quad t < 0$$

$$h_{LP}(t) \neq 0, \quad t < 0$$

The ideal LPF is non-causal and therefore not implementable.

- The third obvious difference is in the time-domain; where $h_{RC}(t)$ decays to zero faster than $h_{LP}(t)$ because of duality

Practical LPF response:



ϵ_1: Passband tolerance
ϵ_2: Stopband tolerance
ω_p : Passband freq
ω_s : Stopband freq.