ECE - 314, Fall 2008
Signals & Systems

Classification of Signals:

Energy Signals: These are signals whose energy is finite, i.e.,

$$\int_a^b |x(t)|^2 \, dt < \infty \qquad \text{or}$$

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$$

Power Signals. These are signals whose average power is finite, i.e.,

$$\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt < \infty \qquad \text{or}$$

$$\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 < \infty$$

Random Signals: These are signals whose values at a specific instant of time are uncertain and belong to the real number line.

Examples

(i) $x(t) = \cos(2\pi f_0 t)$, $t \in \mathbb{R}$

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} \cos^2(2\pi f_0 t)\,dt = \infty$$

$\Rightarrow$ $x(t)$ is not an energy signal

$$P_{ave}^{x} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \cos^2(2\pi f_0 t)\,dt$$

$$P_{ave}^{x} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left(\frac{1+\cos(4\pi f_0 t)}{2}\right) dt$$

$$P_{ave}^{x} = \lim_{T\to\infty} \frac{1}{2T} \cdot T + \lim_{T\to\infty} \frac{1}{2T} \int_{-T/2}^{T/2} \cos(4\pi f_0 t)\,dt$$

$$P_{ave}^{x} = \frac{1}{2} + \lim_{T\to\infty} \frac{1}{2T} \int_{-T/2}^{T/2} \cos(4\pi f_0 t)\,dt$$

If $\quad T = \frac{2\pi}{2f_0} \cdot r, \; r \in I$

then $\quad P_{ave}^{x} = \frac{1}{2}$

If $\quad T \neq \frac{2\pi}{2f_0} \cdot r, \; r \in I$

$$P_{ave}^{x} = \frac{1}{2} + \lim_{T \to \infty} \frac{\sin(4\pi f_0 \frac{T}{2}) \cdot 2}{(4\pi f_0)\, 2T}$$

$$P_{ave}^{x} = \frac{1}{2} + \cancelto{0}{\lim_{T \to \infty} \frac{\sin(2\pi f_0 T)}{4\pi f_0 T}}$$

$\Rightarrow \quad P_{ave}^{x} = \frac{1}{2} < \infty$

$\Rightarrow \quad x(t)$ is a power signal

(2) $$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}} = \frac{4}{3} < \infty$$

$\Rightarrow x[n]$ is a energy signal.

(3) $\quad x(t) \in \{1, -1\}, \quad t \in \mathbb{R}^1,$

$$Pr\{1\} = p$$

$$Pr\{-1\} = 1 - p$$

In fact $x(t), \ t \in \mathbb{R}^1$ is a Bernoulli random variable

$\Rightarrow x(t)$ is a stochastic or random signal.