Problem Set # 6.0
EECE-340, Probability and Statistics
Fall 2002, Department of E.E.C.E.
University of New Mexico, Albuquerque
Assigned : 11/11/02, Due: 11/18/02

Problem 1.0: Pair of Random Variables

Consider a random vector (X, Y) with joint PDF

$$f_{XY}(x,y) = \begin{cases} k & 0 \le x \le 1,\ 0 \le y \le x^2 \\ 0 & \text{otherwise} \end{cases}$$

1. Determine the value k such that $f_{XY}(x,y)$ is a valid joint PDF, i.e., integrates to 1.
2. Find the joint CDF of the random variables (X, Y).
3. Determine the marginal PDF of variables X and Y. Are the random variables independent?
4. Determine the mean and variance of X as well as Y.

Problem 2.0: Joint Statistics

Two zero-mean Gaussian random variables X and Y defined on the same sample space have a covariance matrix of the form:

$$\mathbf{C}_{xy} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

1. Determine the joint statistics, i.e., $r_{xy}, \sigma_{xy}, \rho_{xy}$ associated with these random variables.
2. What is the angle between the random variables θ_{xy}.
3. Determine the marginal PDF's associated with the random variables X and Y.
4. Determine the determinant, the eigenvalues and eigenvectors associated with this covariance matrix. Comment on what is special about the eigenvalues and eigenvectors of this covariance matrix.

Problem 3.0: Transformation of Random Vectors

Let X and Y be two independent Gaussian random variables with zero mean and variance σ^2, i.e., $X, Y \sim N(0, \sigma^2)$ defined on the same sample space. The random variables Z and W are define via

$$\begin{pmatrix} Z \\ W \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

1. Determine the Jacobian associated with the transformed and original random variables. Note that this transformation corresponds to a rotation by an angle θ in Euclidean space.

2. Determine the joint PDF $f_{ZW}(z, w)$ associated with the transformed random variables.

3. Determine the marginal PDF's associated with the transformed random variables Z and W.

4. Determine the mean and variance of the variables Z and W.

Problem 4: Transformations Continued

Let X_1, X_2 and X_3 be independent random variables defined on the same sample space with identical PDF $f_X(x)$, i.e., *independent and identically distributed* (IID) random variables. Determine the CDF and PDF associated with the following transformations:

1. $Y = \min(\max(X_1, X_2), X_3)$
2. $Y = \max(\min(X_1, X_2), X_3)$
3. $Y = \max(X_1, X_2, X_3)$.