

## FOURIER TRANSFORM PAIRS for the DTFT

The index-domain signal is  $x[n]$  for  $-\infty < n < \infty$ ;  
and the frequency-domain values are  $X(e^{j\omega})$  for  $-\pi \leq \omega \leq \pi$ .

Signal: $x[n]$	Fourier Transform (DTFT): $X(e^{j\omega})$
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$
1 $(-\infty < n < \infty)$	$\sum_{r=-\infty}^{\infty} 2\pi \delta(\omega + 2\pi r)$
$e^{j\omega_0 n} \quad (-\infty < n < \infty)$	$\sum_{r=-\infty}^{\infty} 2\pi \delta(\omega - \omega_0 + 2\pi r)$
$\cos(\omega_0 n + \phi) \quad (-\infty < n < \infty)$	$\pi \sum_{r=-\infty}^{\infty} [ e^{j\phi} \delta(\omega - \omega_0 + 2\pi r) + e^{-j\phi} \delta(\omega + \omega_0 + 2\pi r) ]$
$\sin(\omega_0 n + \phi) \quad (-\infty < n < \infty)$	$\pi \sum_{r=-\infty}^{\infty} [ -j e^{j\phi} \delta(\omega - \omega_0 + 2\pi r) + j e^{-j\phi} \delta(\omega + \omega_0 + 2\pi r) ]$
$\frac{\sin \omega_c n}{\pi n} = \text{“sinc” function}$	$X(e^{j\omega}) = \begin{cases} 1, &  \omega  \leq \omega_c \\ 0, & \omega_c <  \omega  \leq \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \leq n < L \\ 0, & \text{otherwise} \end{cases}$ NOTE: pulse length is $L$	$e^{-j\omega(L-1)/2} \frac{\sin(\omega L/2)}{\sin(\omega/2)} = e^{-j\omega(L-1)/2} \text{Dirichlet}(\omega, L)$
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{r=-\infty}^{\infty} \pi \delta(\omega + 2\pi r)$
$a^n u[n] \quad ( a  < 1)$	$\frac{1}{1 - a e^{-j\omega}}$
$(n + 1) a^n u[n] \quad ( a  < 1)$	$\frac{1}{(1 - a e^{-j\omega})^2}$
$\frac{r^n \sin \omega_0 (n + 1)}{\sin \omega_0} u[n] \quad ( r  < 1)$	$\frac{1}{1 - 2r \cos \omega_0 e^{-j\omega} + r^2 e^{-j2\omega}}$
$\sum_{\ell=-\infty}^{\infty} \delta[n - \ell P]$	$\frac{2\pi}{P} \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k/P)$