ECE-439, Fall 2011

Intro to DSP:

Example: DTFT

Consider the sequence:

$$x[n] = -a^n u[-n-1], \quad |a| > 1$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] \exp(-j\omega n)$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{-1} -a^n \exp(-j\omega n)$$

$$X(e^{j\omega}) = \sum_{n=1}^{\infty} -a^{-n} e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=1}^{\infty} -(ae^{j\omega})^{-n}$$

$$= -\left(\sum_{n=0}^{\infty} (ae^{j\omega})^{-n} - 1 \right)$$

$$X(e^{j\omega}) = 1 - \sum_{n=0}^{\infty} r^n, \quad r = a^{-1} e^{-j\omega}$$

$$= 1 - \frac{1}{1-r} = \frac{-r}{1-r}$$

$$X(e^{j\omega}) = \frac{-1}{\frac{1}{r} - 1} = \frac{1}{1 - \frac{1}{r}} = \frac{1}{1 - a e^{j\omega}}$$

MAGNITUDE & PHASE:

$$|X(e^{j\omega})| = \frac{1}{|1 - a\exp(+j\omega)|}$$

$$|X(e^{j\omega})| = \frac{1}{\sqrt{(1 - a\cos\omega)^2 + (a\sin\omega)^2}}$$

$$|X(e^{j\omega})| = \frac{1}{\sqrt{1 + a^2 - 2a\cos\omega}}$$

$$\mathrm{Arg}\,(X(e^{j\omega})) = -\tan^{-1}\left(\frac{-a\sin\omega}{1 - a\cos\omega}\right)$$

$$= \tan^{-1}\left(\frac{a\sin\omega}{1 - a\cos\omega}\right)$$