

ECE - 439, FALL 2011

INTRO TO DSP

EXAMPLE: DTFT OF A STEP

$$X[n] = \underbrace{u[n]}_{V[n]} - \frac{1}{2} + \frac{1}{2}$$

Taking the DTFT on both sides:

$$X(e^{j\omega}) = V(e^{j\omega}) + \frac{1}{2} 2\pi \sum_{r=-\infty}^{\infty} \delta(\omega + 2r\pi)$$

$$X(e^{j\omega}) = V(e^{j\omega}) + \pi \sum_{r=-\infty}^{\infty} \delta(\omega + 2r\pi)$$

$$V(e^{j\omega}) = \sum_{n=-\infty}^{-1} \left(-\frac{1}{2}\right) \exp(-j\omega n) \\ + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \exp(-j\omega n)$$

$$V(e^{j\omega}) = \left(-\frac{1}{2}\right) \sum_{m=1}^{\infty} \exp(j\omega m) \\ + \frac{1}{2} \sum_{n=0}^{\infty} \exp(-j\omega n)$$

$$V(e^{j\omega}) = \left(-\frac{1}{2}\right) \left( \frac{1}{1 - \exp(j\omega)} - 1 \right) \\ + \left(\frac{1}{2}\right) \left( \frac{1}{1 - \exp(-j\omega)} \right)$$

$$V(e^{j\omega}) = \left(-\frac{1}{2}\right) \left(\frac{e^{j\omega}}{1-e^{j\omega}}\right) + \frac{1}{2} \left(\frac{1}{1-e^{-j\omega}}\right)$$

$$V(e^{j\omega}) = \left(-\frac{1}{2}\right) \left(\frac{1}{e^{-j\omega}-1}\right) + \left(\frac{1}{2}\right) \left(\frac{1}{1-e^{-j\omega}}\right)$$

$$V(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}$$

$$X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}} + \pi \sum_{r=-\infty}^{\infty} \delta(\omega + 2r\pi)$$

$\underbrace{\hspace{2cm}}$   
 $\downarrow$   
 continuous part

$\downarrow$   
 Impulsive part