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Digital Signal Processing, EECE-539

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Abstract

This is intended to be a guide for review of material relating to discrete–time signals/systems from EECE-314 which is a prerequisite for this course. I will just briefly overview material in class in these areas but you are expected to know this material. **If you feel that you do not know this material consult the instructor as to whether the course is appropriate for you**. Further material in this course will build on these concepts:

- Characterization of discrete–time signals/systems
- Difference equations
- Impulse and step response.
- Linear convolution sum.
- Nyquist sampling theorem.
- DTFT and frequency response.
- Z transforms and system functions.

Problem 0.1: System Characterization

Determine whether or not the following statements are true (T) or false (F). Justify your answers properly.

1. The cascade of two linear systems is a linear system.

2. Discrete–time LTI operators $\mathbf{L}_1$ and $\mathbf{L}_2$ always commute.

3. The cascade of a causal system with a non causal system is necessarily causal.

4. Causal systems are necessarily stable.

5. A system with an impulse response that has finite energy is a BIBO stable system.

Problem 0.2: System functions

A discrete–time, LTI system has a system function $H(z)$ of the form:

$$H(z) = \frac{2}{(1 - 0.5z^{-1})(1 - 2z^{-1})}, \quad z \in \mathbf{C}^1.$$

1. Determine the input–output difference equation satisfied by the system specified above.

2. If the system is known to be stable then determine the *region of convergence* (ROC) of the system function $H(z)$. Plot this region in the complex Z plane. Determine the corresponding impulse response $h[n]$ of this system.

3. Determine the initial value of the impulse response $h[n]$ and the DC spectral value of $h[n]$ from the system function $H(z)$.

4. Determine the system function $H_{\text{inv}}(z)$, of the system that undoes the effect of the system $H(z)$ and the ROC of $H_{\text{inv}}(z)$.

Problem 0.3: Frequency Response

The input and the output of discrete–time LTI system satisfy the following input-output difference equation:

$$y[n] \ = \ 0.5y[n-1] + x[n].$$

1. Determine the system function $H(z)$ of this system and the corresponding ROC of $H(z)$.

2. Determine the impulse response $h[n]$ of this system.

3. Determine the frequency response, $H(e^{j\omega})$ of this system. How is this related to the system function $H(z)$?

4. Determine the magnitude, phase, and group-delay of this frequency response. Plot its magnitude.

Problem 0.4: Linear Convolution

The signal $x[n] = u[n]$ is the input signal to a discrete–time LTI system that has an impulse response given by $h[n] = (0.5)^n u[n]$. With regards to this system:

1. Compute the output of the system for the input signal given above by evaluating the convolution sum.

2. Compute the output of this system for this input without evaluating the convolution sum using Fourier analysis.

3. What is the output of this system if the input signal to this system is $x_1[n] = 2x[n-1]$ instead.

4. What is the output of this system if the input signal to this system is $x_1[n] = x[n] - x[n-1]$ instead.

Problem 0.5: Nyquist Sampling Theorem

Let $x_c(t)$ be a continuous–time signal whose Fourier spectrum $X_c(j\Omega)$ is bandlimited to the interval $[-\frac{\pi}{T_s}, \frac{\pi}{T_s}]$. This signal is sampled using *zero-order hold* via a rectangular pulse train with pulse–width τ and duty–cycle $\frac{\tau}{T_s}$. With respect to this sampling system:

1. Express the rectangular pulse–train $p(t)$ in terms of its Fourier series representation.

2. Express the spectrum of the quasi-digital-quasi-analog signal, $X_s(j\Omega)$ in terms of the spectrum $X_c(j\Omega)$.

3. How should the sampling frequency f_s of this A/D system be chosen so that there is no loss of information during the sampling process?

4. Can we reconstruct the continuous–time signal $x_c(t)$ from its discrete–time samples $x[n]$ and if so how?