ECE-439, FALL 2011

INTRO TO DSP

Example : Power Series Expansion

Consider a Z-transform expression of the form :

$$X(z) = \ln(1+az^{-1}) \quad , \quad |z| > |a|$$

Consider the Taylor series expansion for $\ln(1+x)$ about $|x| < 1$:

$$\ln(1+x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots\cdots - \frac{x^n}{n} + \cdots$$

or

$$\ln(1+x) = -\sum_{i=1}^{\infty} \frac{x^i}{i}$$

Replacing $x = az^{-1}$:

$$\ln(1+az^{-1}) = -\sum_{i=1}^{\infty} \frac{(az^{-1})^i}{i}$$

$$\ln(1+az^{-1}) = -\sum_{i=1}^{\infty} \frac{a^i}{i} z^{-i}$$

$$\ln(1+az^{-1}) = \sum \frac{-a^i}{i} z^{-i}$$

Let us define a sequence:

$$x[n] = \begin{cases} -\dfrac{a^n}{n}, & n \geq 1 \\ 0, & n \leq 0 \end{cases}$$

The Power-series expansion is written as:

$$\ln(1+az^{-1}) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

which can be recognized as the Z-transform of $x[n]$.

We therefore obtain the Z transform pair:

$$\ln(1+az^{-1}) \underset{Z}{\overset{Z^{-1}}{\rightleftharpoons}} x[n] = -\frac{a^n}{n} u[n-1]$$

$|z| > |a|$

Here we have obtained the Inverse Z transform by interpreting the transform as a power series expansion. that converges in a specified region in the z-plane