ECE-495, FALL 2011

INTRO TO DSP

Examble: Rectangular Pulse Sampling

Suppose the impulse train used in the derivation of the sambling theorem is replaced with a realistic rectangular pulse train with duty cycle $\frac{\tau}{T_s}$, i.e.,

$$p_{T_s}(t) = \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t}{T_s} - k\right)$$

This is a periodic waveform with a Fourier series representation:

$$p_{T_s}(t) = \sum_{k=-\infty}^{\infty} c[k] \exp\left(j \frac{2\pi}{T_s} k t\right)$$

$$c[k] = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} p_{T_s}(t) \exp\left(-j \frac{2\pi}{T_s} k t\right) dt$$

Substituting the expression for $p_{T_s}(t)$:

$$c[k] = \frac{1}{T} \int_{-\tau/2}^{\tau/2} \exp\left(-j \frac{2\pi}{T_s} k t\right) dt$$

$$= \left(\frac{\tau}{T_s}\right) Sa\left(\frac{k\pi\tau}{T_s}\right)$$

The Fourier transform of the rectangular pulse train is

$$P_{T_s}(j\Omega) = 2\pi \sum_{k=-\infty}^{\infty} c[k]\, \delta(\Omega - k\tfrac{2\pi}{T_s})$$

Substituting this expression into the derivation of the alias - spectrum

$$X_s(j\Omega) = \sum_{k=-\infty}^{\infty} c[k]\, X_c(j(\Omega - k\tfrac{2\pi}{T_s}))$$

This expression is indicative of several important observations:

(1) The desired term is still the $k=0^{th}$ term in the sum

(2) The gain of the LPF in the D/A is $\frac{T_s}{\tau}$

(3) Sampling Theorem still holds with a minor modification.