ECE - 439, Fall 2011

INTRO TO DSP :

Example : Bilateral Z transform

Consider the discrete sequence :

$$x[n] = a^{|n|} = \begin{cases} a^{n}, & n \geq 0 \\ a^{-n}, & n < 0 \end{cases}$$

$(|a| < 1)$

The Zee transform of this sequence is given by :

$$X(z) = \sum_{n=-\infty}^{n=-1} a^{-n} z^{-n} + \sum_{n=0}^{\infty} a^{n} z^{-n}$$

$$X(z) = \sum_{m=1}^{\infty} (az)^{m} + \sum_{n=0}^{\infty} (az^{-1})^{n}$$

$$X(z) = -1 + \frac{1}{1-az} + \frac{1}{1-az^{-1}}$$

Converges for $|az| < 1$ or $|z| < \frac{1}{|a|}$

Converges for $|az^{-1}| < 1$ or $|z| > |a|$

Combining the two expressions.

$$X(z) = \frac{+az}{1-az} + \frac{1}{1-az^{-1}}$$

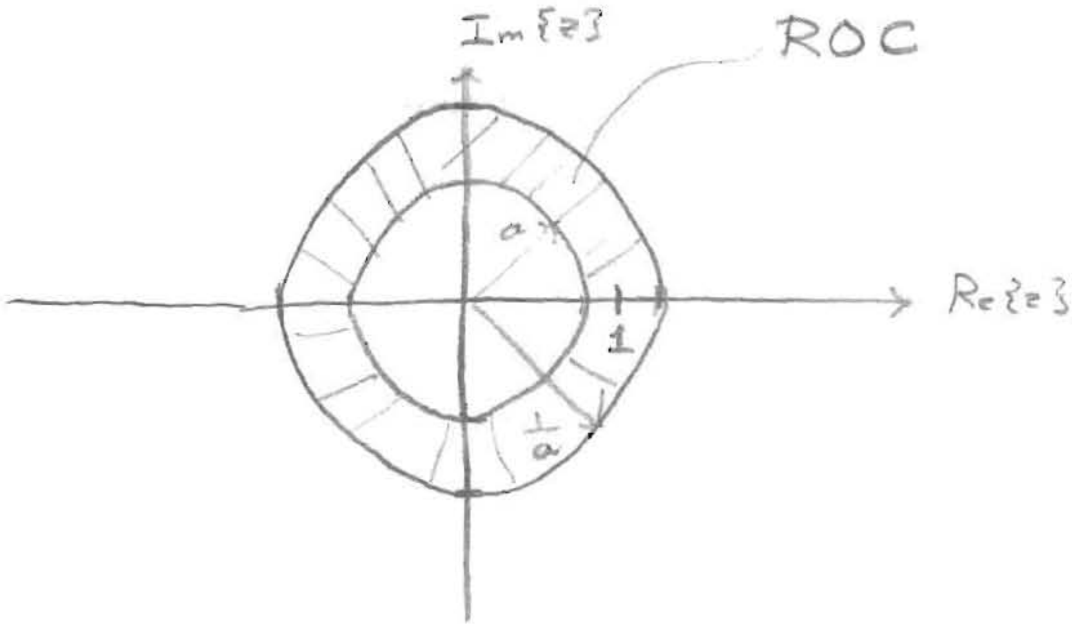
$$X(z) = \frac{+az - a^2 + 1 - az}{(1-az^{-1})(1-az)}$$

$$X(z) = \frac{1-a^2}{(1-az^{-1})(1-az)}$$

The corresponding ROC is given by:

$$\{z \ni |z| > |a|\} \cap \{z \ni |z| < \frac{1}{|a|}\}$$

$$\equiv \{z \ni |a| < |z| < \frac{1}{|a|}\}$$



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