Problem Set #5
EECE-495, Spring 2001
Signal Processing Using MATLAB
MATLAB Assignment
Date Assigned: 03/23/2001
Date Due: 03/30/2001

Background

The technique of coding information bits onto the frequency of a sinusoid, i.e., *frequency based coding* is one that finds application in *frequency shift keying* (FSK) based digital communications. In fact a dual–tone based frequency coding system has been adopted as the *ATT Touchtone Standard*. In this exercise we will simulate the frequency based coding and decoding of signals. For simplicity sake, we will restrict our discussion to the digits between 0 and 4. Each of these digits is assigned a distinct sinusoidal frequency as described in fig. (1).

The process of frequency based coding is embodied in the *instantaneous frequency* (IF) signal that assumes the form:

$$\omega_i(t) = \sum_{k=0}^{K-1} \omega_c[k] \mathrm{rect}\left(\frac{t - kT_d}{T_d}\right),$$

where K is the number of digits in the coding alphabet and T_d is the duration of the digit signal. Of course in situations where we are under bandwidth constraints other smooth pulse shaping functions can be used instead of the rectangular signal. For simplicity sake, we will restrict our discussion to the rectangular signal. The frequency coded signal is therefore a *frequency modulated* (FM) signal of the form:

$$s(t) = \cos\left(\int_{-\infty}^{t} \omega_i(\tau) d\tau\right).$$

The decoding process can therefore be considered as a FM *demodulation* problem. Although a number of demodulation algorithms exist we will restrict our discussion to the *Hilbert transform demodulation* (HTD) algorithm. The Hilbert transform of a signal $s(t)$ is defined via:

$$s_H(t) \; = \; s(t) * \frac{1}{\pi t} \; = \; \int_{-\infty}^{\infty} \frac{1}{\pi \tau} s(t-\tau) d\tau.$$

For the specific case where the signal is a pure tone, i.e., $s(t) = A_m \cos(\omega_m t)$, the Hilbert transform is a *quadrature version*, i.e., a 90° phase shifted version

Digit	0	1	2	3	4
Frequency	$\frac{\pi}{2}$	$\frac{\pi}{2}+\frac{\pi}{200}$	$\frac{\pi}{2}+\frac{\pi}{100}$	$\frac{\pi}{2}+\frac{3\pi}{200}$	$\frac{\pi}{2}+\frac{4\pi}{200}$

Figure 1: Frequency based signal coding: mapping from the digits to digital sinusoidal frequencies. The sampling frequency associated with the digits is $f_s = 10$ kHz and the duration of a single digit is 0.1 sec.

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of the signal $s(t)$, i.e., $s_H(t) = A_m \sin(\omega_m t)$. For the case where the frequency coded signal is narrowband and the frequency of the sinusoid is slowly time–varying, i.e., $\omega_i(t)$ varies slowly with respect to time:

$$s(t) = \cos(\phi_i(t)) = \cos\left(\int_{-\infty}^{t} \omega_i(\tau)d\tau\right),$$

the Hilbert transform is approximately its quadrature version:

$$s_H(t) \approx \sin(\phi_i(t)) = \sin\left(\int_{-\infty}^{t} \omega_i(\tau)d\tau\right).$$

The instantaneous phase of the signal can then be estimated via:

$$\hat{\phi}_i(t) = \tan^{-1}\left(\frac{s_H(t)}{s(t)}\right).$$

The instantaneous frequency of the signal is then estimated as the derivative of the instantaneous phase estimate:

$$\hat{\omega}_i(t) = \frac{s(t)\dot{s_H}(t) - s_H(t)\dot{s}(t)}{s^2(t) + s_H^2(t)},$$

where the dot on the top of a signal denotes the time derivative of the signal. The instantaneous frequency is averaged over the duration of the digit to obtain an estimate of the corresponding frequency:

$$\hat{\omega}_c[k] \approx \frac{1}{T_d}\int_{(k-1)T_d}^{kT_d} \hat{\omega}_i(t)dt, \quad 0 \le k \le 4.$$

For implementing the decision device that maps the estimated sinusoidal frequency back to digits, assume that each of the digits are equally likely and place the decision boundary at the midpoint between the sinusoidal frequencies on either side of the boundary. The envisioned decision device rounds the estimated sinusoidal frequency to the nearest entry in the table of fig. (1) and assigns the estimated sinusoidal frequency a corresponding digit.

Program Outline

1. Write a matlab function `freqcode.m` with synopsis:

   ```
   s = freqcode(digits,f_s,T_d)
   digits: sequence of 5 numbers containing digits 0-4
   f_s : sampling frequency (Hz)
   T_d: duration of the digit (sec)
   ```

   that generates a frequency coded input signal from digits with specified duration and sampling period. The coding procedure is comprised of three operations: (a) mapping the entered digits to sinusoidal frequencies, (b) creating the IF signal with information coded on it and (c) integrating the IF signal to obtain the frequency coded signal.

2. Write a matlab function `freqdecod.m` with synopsis:

   ```
   digit = freqdecod(s)
   s: frequency coded input signal
   digit: output digit containing digits 0-4
   ```

   that implements the decoding procedure comprised of (a) hilbert transform demodulation, (b) time-averaging to estimate the sinusoidal frequencies and (c) the decision making device that maps the estimated frequencies to digits and returns a vector of the 5 digits entered.

3. MATLAB hints: Use the matlab program `hilbert.m` to implement the HTD algorithm. Use the matlab program `diff.m` to implement the differentiation operation needed in decoding operation and use the matlab program `cumsum.m` to implement the integration operation that relates the instantaneous frequency and phase. Use the matlab program `mean.m` to implement the averaging operation needed to estimate the sinusoidal frequencies.