PS #1 , Spring 2001
Signal Processing Using MATLAB, EECE-495
Instructor: Balu Santhanam
MATLAB Assignment
Date Assigned: 01/24/2001
Date Due: 02/01/2001

Background

A typical lowpass, second–order, continuous–time, LTI system has a transfer function of the form

$$H(s) = \frac{\omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}.$$

The parameter ω_o is refered to as the resonance frequency. The parameter Q is the quality factor of the system. The corresponding frequency response, $H(\omega)$ is obtained by evaluating the above along $s = j\omega$ as :

$$H(\omega) = \frac{\omega_o^2}{\omega_o^2 - \omega^2 + j\frac{\omega\omega_o}{Q}}.$$

The poles of this system, i.e., the roots of the denominator of $H(s)$ are given by:

$$s_{1,2} = -\frac{\omega_o}{2Q} \pm \sqrt{\left(\frac{\omega_o}{2Q}\right)^2 - \omega_o^2}.$$

Specifically to obtain real coefficients, the location of these poles have to be at complex conjugate locations and can be represented in terms of polar coordinates as:

$$s_{1,2} = r\left(\cos\psi + j\sin\psi\right) = re^{j\psi}.$$

Comparing the real and imaginary parts of the poles and solving for the parameters ω_o, Q we obtain:

$$\omega_o = r\ ,\ \ Q = -\frac{1}{2\cos\psi}.$$

The 3dB bandwidth of this system is defined by :

$$\beta = \frac{\omega_o}{Q} = -2r\cos\psi.$$

The damping factor of this system is then defined via :

$$\zeta = -\frac{1}{2Q} = \cos\psi.$$

Problem Outline

Write a matlab script `filt.m` that incorporates the following:

1. Asks the user to input the coefficients $[a, b, c]$ of a quadratic polynomial $f(s) = as^2 + bs + c$ in the complex variable s with $a \neq 0$.

2. Computes the roots of the quadratic, i.e., solutions to $f(s) = 0$ after normalizing by a.

3. Computes the quality factor Q, damping ratio ζ and bandwidth β associated with the roots $s_{1,2}$.

4. Display the computed results on the screen in short floating point format.

5. Plot the quadratic $f(s)$ for $s \in [-2.0, 2.0]$ with increments of $s = 0.25$. Label the xaxis, yaxis and plot properly. Switch the grid on and use *Helvetica 16* for the fonts on the graph. Save the plot to the file `filt.eps`.

6. Tabulate the roots, the real and imaginary parts of the roots, the magnitude and phase associated with each root, the corresponding Q, ζ and β and save the results in the diary file `filt`.

7. Save the variables $s_{1,2}, Q, \zeta, \beta$ in the matlab file `filt.mat`.