PS # 4.0, Spring 2002
Engineering Problem Solving Using MATLAB
EECE-495/595, University of New Mexico
Instructor: Balu Santhanam
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1 Background

Let X be a continuous random variable defined over a sample space $\mathbf{S}$ with a specified CDF $F_X(x)$. MATLAB has two buit-in random number generators `rand.m` and `randn.m` that generate random numbers that are either unifromly distributed or gaussian distributed. Our goal in this section is to come up with an algorithm to generate samples of a random variable that have the specified CDF. For this purpose, let us first consider the random variable $\mathbf{Y}$ defined via

$$\mathbf{Y} = F_X(\mathbf{X}).$$

From the properties of the CDF for a continuous RV we know that the CDF is a monotonically increasing function between 0 and 1,i.e.,

$$x_1 \leq x_2 \Longleftrightarrow F_X(x_1) \leq F_X(x_2), \quad F_X(x) \in [0,1].$$

and from the fact that X is a continuous RV we can show that $F_X(x)$ is a uniformly continuous function and therefore the transformation $F_X(\mathbf{X})$ is a bijective mapping from $\mathbf{R}$ to the interval $[0,1]$. Hence a unique inverse $F_X^{-1}(x)$ for the CDF exists. Using the set equivalence method the CDF of the transformed variable $\mathbf{Y}$ we obtain:

$$F_Y(y) = \Pr(Y \leq y) = \Pr(X \leq F_X^{-1}(y)) = F_X(F_X^{-1}(y)) = y, y \in [0,1].$$

The PDF of the random variable obtained by taking the derivative of the CDF in the above expression is therefore :

$$f_Y(y) = \begin{cases} 1 & y \in [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

This implies that the RV generated from the transformation $\mathbf{Y} = F_X(\mathbf{X})$ is uniform on the interval $[0,1]$, i.e., $Y \sim U([0,1])$. This fact forms the basis for the algorithm to generate samples of a RV with a specified CDF:

- Generate N samples of a random variable $Z \sim [0,1]$ using the function `rand.m`.
- Generate the inverse CDF $F_X^{-1}(x)$ using a built–in MATLAB function or a written function.
- Apply the transformation F_X^{-1} on the samples of Z.

2 Problem Statement

1. Write a MATLAB program `randgen.m` with the synopsis:

```
r = randgen(N,para,type)
SYNOPSIS:
N    : number of samples needed
para : vector containing parameters of distribution
type : string containing the type of distribution needed
       among the options: (a) Gaussian, (b) Uniform,
       (c) Exponential (d) Laplacian, (e) Beta,
       (f) Gamma, (g) Weibull
```

2. Use appropriate error checking in the function.

3. You can make use of the built-in MATLAB random number generators `rand.m` and `randn.m` and the MATLAB inverse CDF functions.

4. Use 4 letter strings for the type of random variable.

5. Test the program for the following, $N = 2000$: (a) Exponential with $\lambda = 1$, (b) Gamma with parameters $m = 0.5$ and $n = 1$, (c) Uniformly distributed on $[2, 3]$. Verify the result by plotting the histogram.