Problem Set # 7.0
Signal Processing Exercises in MATLAB
Instructor: Balu Santhanam
EECE–495, Spring 2001
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In problem set # 4.0, we designed a binary detector for a AWGN channel. The performance characteristics of this receiver were discussed in relation to a binary hypothesis testing problem. In this exercise, we will study the effects of random flat channel fading on the *maximum likelihood detector* (MLD). In addition we will simulate a predetection selective diversity combining scheme that will mitigate the effects of flat channel fading.

Background

The binary hypothesis testing problem in the AWGN channel is:

$$\begin{aligned} \mathbf{H}_0 &: \quad s = \mu_1 + n \\ \mathbf{H}_1 &: \quad s = \mu_2 + n, \end{aligned}$$

In the presence of flat channel fading the corresponding hypothesis testing problem becomes:

$$\begin{aligned} \mathbf{H}_0 &: \quad s = \alpha\mu_1 + n \\ \mathbf{H}_1 &: \quad s = \alpha\mu_2 + n, \end{aligned}$$

where α is either a Rayleigh faded random variable. The *bit energy to noise ratio* (BENR) in this case is given by:

$$\gamma_b = \frac{\alpha^2 E_b}{N_o}.$$

In this case this quantity itself becomes a exponential random variable with PDF:

$$f_{\gamma_b}(x) = \frac{1}{\gamma_c} \exp\left(-\frac{x}{\gamma_c}\right) U(x),$$

where $\gamma_c = \frac{E_b}{N_o}$. Conditioned on a particular BENR γ_b the probability of decision error is:

$$\Pr\left(\epsilon | \gamma_b = x\right) = Q\left(\sqrt{2x}\right).$$

The average probability of decision error can then be computed as:

$$\Pr(\epsilon) = \frac{1}{\gamma_c} \int_0^\infty Q\left(\sqrt{2x}\right) \exp\left(-\frac{x}{\gamma_c}\right) dx.$$

Using integration by parts and Lebnitz rule the integral can be evaluated as:

$$\Pr(\epsilon) = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_c}{1+\gamma_c}}\right).$$

In the absence of fading the corresponding expression for the average probability of decision error is:

$$\Pr(\epsilon) = Q(\sqrt{2\gamma_c}).$$

To achieve a specified level of performance of $\Pr(\epsilon) = 10^{-5}$, it requires a BENR of 9.6 dB in the absence of fading. In the presence of fading it requires a BENR of 44 dB. This means that there is significant deterioration in detector performance in the presence of flat fading. We therefore are in need of techniques that will decrease the $\Pr(\epsilon)$ without having to boost transmit power to high levels.

The common theme in all of these diversity schemes is that multiple copies of the signal are transmitted as opposed to just one transmit signal. At the receiver end the output from the diversity branches is combined in different ways so that the effective bit error rate is reduced. One such diversity signaling scheme is the *predetection selective combining* technique. The idea behind this diversity method is to choose the diversity branch that gives you the largest BENR and to use the decision made by the chosen branch. In other words the output decision variable is just the maximum statistic, i.e.,

$$\gamma_b^{\text{eff}} = \max\{\gamma_1, \gamma_2\},$$

where γ_1 and γ_2 are independent and identically distributed exponential random variables with mean $\gamma_c = \frac{E_b}{N_o}$. This type of diversity receiver essentially employs majority logic to obtain a decision on the data bit. Specifically let us look at the two branch diversity receiver. The PDF of the decision variable can be evaluated as:

$$f_{\gamma_b^{\text{eff}}}(x) = 2F_{\gamma_b}(x)f_{\gamma_b}(x) = 2(1 - e^{-\frac{x}{\gamma_c}})e^{-\frac{x}{\gamma_c}}U(x).$$

The average BENR for this receiver is given by:

$$E\{\gamma_b^{\text{eff}}\} = \int_0^\infty x f_{\gamma_b^{\text{eff}}}(x)dx = \frac{3}{2}\gamma_c.$$

The average BENR is therefore boosted and maximum diversity gain is obtained in the two branch case, while diminishing returns are obtained upon addition of more branches. The probability of decision error for this majority logic receiver can be shown to be:

$$\Pr(\epsilon) = \frac{1}{2}\left(1 - 2\sqrt{\frac{\gamma_c}{1+\gamma_c}} + \sqrt{\frac{\gamma_c}{2+\gamma_c}}\right).$$

Program Outline

1. Write a MATLAB script `rfade.m` that simulates a flat Rayleigh fading channel. Use the functions `bingen.m` and `awn.m` written before. The variance σ_f^2 of the amplitude fade α determines the rate at which the envelope fades.

2. Study the conventional AWGN binary detector `bindet.m` in the presence of Rayleigh fading.

3. Compare the results obtained to the theoretical results.

4. Write a MATLAB function `scdiv.m` that implements the majority logic receiver and study its performance.

5. Compare the performance curve obtained to the theoretical results.