PS #3 , Spring 2015

Signal Processing Using MATLAB, ECE-495/595

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Date Assigned: 03/03/2015

Date Due: After spring break

In this assignment, we will look at the operations that change the sampling rate of a sequence and use them to implement a transmultiplexer. The inputs in the exercise are voice or audio signals that are clearly discernible.

Background

For a two channel *perfect reconstruction* (PR) TMUX, the design equations are given by:

$$\mathbf{E}(z)\mathbf{\Gamma}(z)\mathbf{R}(z) = cz^{-n_o}\mathbf{I}_{2\times 2},$$

where $\mathbf{R}(z)$ and $\mathbf{E}(z)$ are the type-II and type-I polyphase matrices associated with the synthesis and analysis sections respectively and $\mathbf{\Gamma}(z)$ is defined via:

$$\mathbf{\Gamma}(z) = \begin{pmatrix} 0 & 1 \\ z^{-1} & 0 \end{pmatrix}$$

Suppose the analysis filters are given by:

$$\begin{aligned} H_o(z) &= 1 + z^{-1} + z^{-2} - z^{-3} \\ H_1(z) &= 1 + z^{-1} - z^{-2} + z^{-3}. \end{aligned}$$

In this case, the analysis polyphase matrix is given by:

$$\mathbf{E}(z) = \begin{pmatrix} 1 + z^{-1} & 1 - z^{-1} \\ 1 - z^{-1} & 1 + z^{-1} \end{pmatrix}$$

Substituting this into the PR-TMUX equations we have:

$$\mathbf{R}(z) = cz^{-n_o}\mathbf{\Gamma}^{-1}(z)\mathbf{E}^{-1}(z).$$

or

$$\mathbf{R}(z) = cz^{-n_o}\frac{1}{-z^{-1}}\begin{pmatrix} 0 & -1 \\ -z^{-1} & 0 \end{pmatrix}\mathbf{E}^{-1}(z).$$

Substituting the expression for $\mathbf{E}(z)$ into this expression:

$$\mathbf{R}(z) = cz^{-n_o}\frac{1}{z^{-1}}\begin{pmatrix} 0 & 1 \\ z^{-1} & 0 \end{pmatrix}\frac{1}{4z^{-1}}\begin{pmatrix} 1 + z^{-1} & -1 + z^{-1} \\ -1 + z^{-1} & 1 + z^{-1} \end{pmatrix}.$$

Choosing $c = 4, n_o = 2$, we obtain:

$$\mathbf{R}(z) = \begin{pmatrix} -1 + z^{-1} & 1 + z^{-1} \\ z^{-1} + z^{-2} & -z^{-1} + z^{-2} \end{pmatrix}$$

The corresponding synthesis filters are given by:

$$\begin{aligned} F_o(z) &= -z^{-1} + z^{-3} + z^{-2} + z^{-4} \\ F_1(z) &= z^{-1} + z^{-3} - z^{-2} + z^{-4}. \end{aligned}$$

Arbitrary Filters

Using arbitrary filters in the TMUX will result in both distortion and cross-talk at the two outputs. Using FIR filters of order $L = 5$ with random impulse response coefficients implement the TMUX using speech or audio waveforms and comment on the distortions heard in the outputs. Towards the implementation, use `upsample.m` and `downsample.m` to implement the upsampler and downsampler and use the function `filter.m` to implement the filtering operations. You can use the function `spectrum.m` to calculate the spectrogram of the relevant audio waveforms.

PR-TMUX Design

Using the filters designed in the background section, implement the two-channel TMUX and comment on the audio outputs of the system. Do you observe any distortion?

Channel Distortion

The analysis in the background section was done in the absence of a channel. Suppose we introduce a channel that has two parts: (a) AWGN noise and (b) frequency selective distortion. Using the channel described in the class $C(z) = 1 + 0.2z^{-1}$, comment on the outputs in this case.