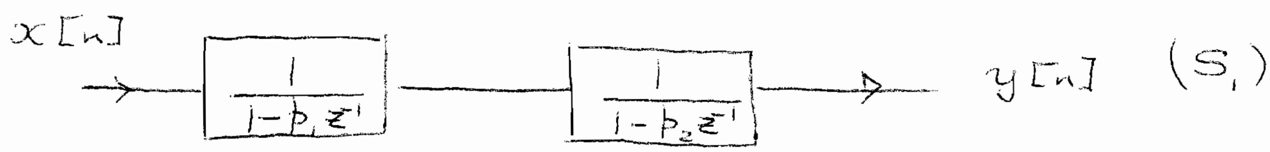
ECE-539, SPRING 2009
Digital Signal Processing

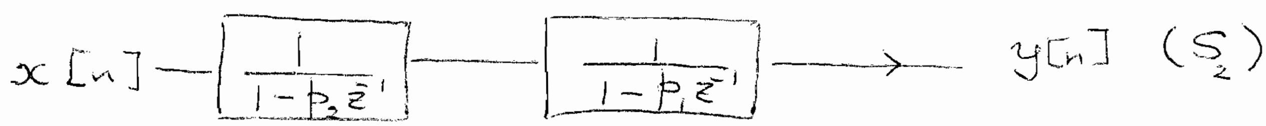
Example : Cascade Form

Consider a causal, LTI system with system function

$$H(z) = \frac{1}{(1-p_1 z^{-1})(1-p_2 z^{-1})}, \quad |z| > \max\{p_1, p_2\}$$

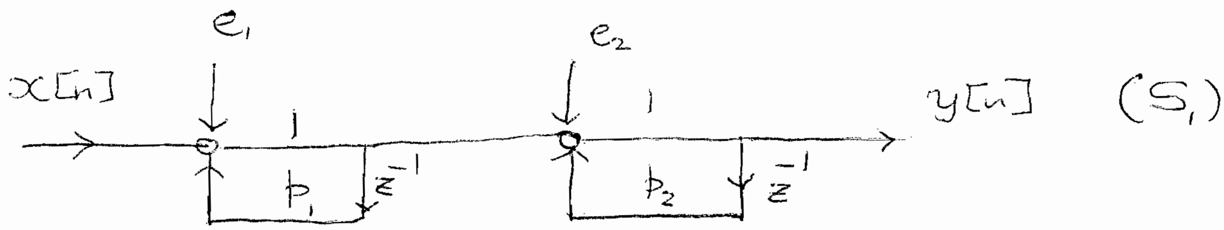
The two cascade structures:

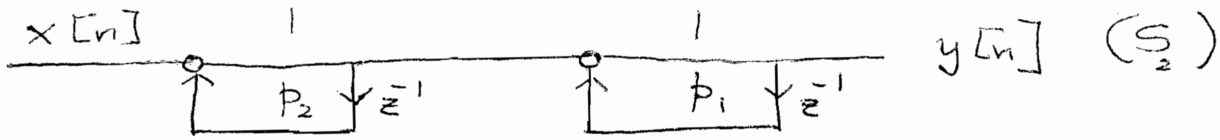




Suppose we quantize the sum of products to (B+1)-bits

Finite Precision Model





S_1 :

$$\sigma_f^2 = \sigma_e^2 \sum_{n=0}^{\infty} (p_2)^{2n} u[n]$$
$$+ \sigma_e^2 \sum_{n=0}^{\infty} h^2[n]$$

S_2 :

$$\sigma_f^2 = \sigma_e^2 \sum_{n=0}^{\infty} (p_1)^{2n} u[n]$$
$$+ \underbrace{\sigma_e^2 \sum_{n=0}^{\infty} h^2[n]}_{T_1}$$

Ignoring the common term :

$$T_1 = \begin{cases} \dfrac{\sigma_e^2}{1-p_2^2}, & S_1 \\ \dfrac{\sigma_e^2}{1-p_1^2}, & S_2 \end{cases}$$

For $|p_1|, |p_2| < 1$

If $|p_1|^2 < |p_2|^2$, i.e., $|p_1| < |p_2|$

$\Rightarrow -p_1^2 > -p_2^2$

$\Rightarrow 1 - p_1^2 > 1 - p_2^2$

$\Rightarrow \dfrac{1}{1-p_1^2} < \dfrac{1}{1-p_2^2}$

$\Rightarrow \dfrac{\sigma_e^2}{1-p_1^2} < \dfrac{\sigma_e^2}{1-p_2^2}$

$\Rightarrow$ S_2 produces quantization noise with smaller average power

- Placing the subsystem with pole closer to UC at the front reduces the quantization noise power at the output.

- For higher-order sections, subsections with larger Q are placed at the front.