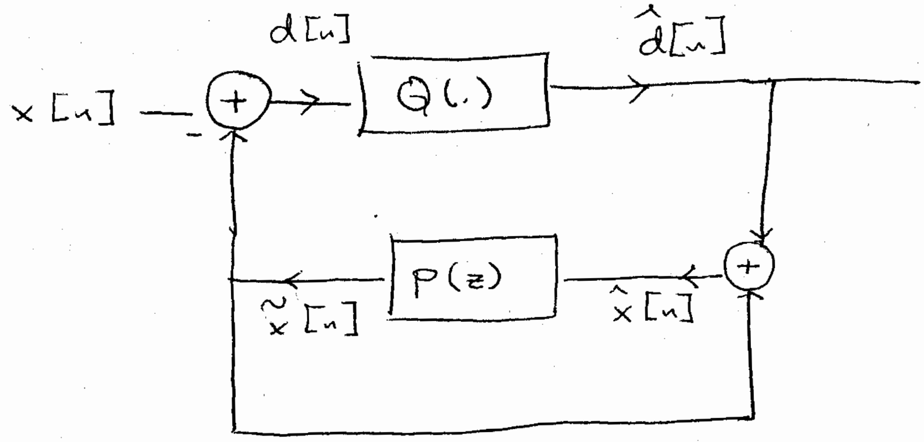
ECE-539, SPRING 06

Differential Quantization



$$(1) \quad d[n] = x[n] - \tilde{x}[n] \quad \text{(Prediction Error)}$$

$$(2) \quad \hat{d}[n] = d[n] + e[n] \quad \text{(Uniform Quantization)}$$

$$(3) \quad \hat{x}[n] = \tilde{x}[n] + \hat{d}[n]$$

From (1):

$$\tilde{x}[n] = x[n] - d[n]$$
$$\hat{d}[n] = d[n] + e[n]$$

adding the Eq's together

$$\tilde{x}[n] + \hat{d}[n] = x[n] + e[n] = \hat{x}[n]$$

$\Rightarrow$ The difference between $\hat{x}[n]$ and $x[n]$ is the same as the difference between $\hat{d}[n]$ and $d[n]$

$\Rightarrow$ Quantization of $d[n]$ is equivalent quantization of $x[n]$

$\Rightarrow$ Quantization of $d[n]$ uncoupled with the predictor operation

Salient Features

- Oversampling Input Signal should result in significant temporal coherence in the samples of $x[n]$

- Significant temporal coherence implies that by an appropriate choice of predictor coefficients, the variance of the prediction error $d[n] = x[n] - \tilde{x}[n]$ can be made small in comparison to the variance of $x[n]$

- Quantization of prediction error or difference signal $d[n]$

- Knowledge of predictor coefficients along with prediction error sequence regenerates $\hat{x}[n]$ (over-head)

- Dynamic range of prediction error can be made small by design of a predictor of a suitable order

Performance Metric

$$SNR_o = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right)$$

$$= 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_d^2}\cdot\frac{\sigma_d^2}{\sigma_e^2}\right)$$

$$SNR_o = \underbrace{10\log_{10}\left(\frac{\sigma_x^2}{\sigma_d^2}\right)}_{\text{Term governed by predictor}} + \underbrace{10\log_{10}\left(\frac{\sigma_d^2}{\sigma_e^2}\right)}_{\text{Term depending on quantizer}}$$

First-order predictor example:

$$\tilde{x}[n] = c\hat{x}[n-1] = c(\tilde{x}[n-1] + d[n-1] + e[n-1])$$

or

equivalently

$$\tilde{x}[n] = c\,x[n-1]$$

$$c_{opt} = \frac{r_{xx}(1)}{r_{xx}(0)} = \rho_{xx}(1)$$

$$J_{min} = \sigma_x^2(1-\rho_{xx}^2(1))$$

or $$\frac{\sigma_x^2}{\sigma_d^2} = \frac{1}{1-\rho_{xx}^2(1)}$$

$$SNR_o = -10\log_{10}(1-\rho_{xx}^2(1)) + 10\log_{10}\left(\frac{\sigma_d^2\cdot 12}{\Delta^2}\right)$$

For $\rho_{xx}(1) = 0.8 - 0.9 \Longrightarrow$ 7.21dB improvement

For fixed $SNR_o \longrightarrow$ reduce bit resolution by 1 bit