Example:

$$K_1 = \frac{1}{4}, \quad K_2 = \frac{1}{2}, \quad K_3 = \frac{1}{3}$$

$$A_0(z) = B_0(z) = 1$$

$$A_1(z) = 1 + K_1 z^{-1} = 1 + \tfrac{1}{4} z^{-1}$$

$$B_1(z) = z^{-1}(1 + \tfrac{1}{4} z) = \tfrac{1}{4} + z^{-1}$$

$$\begin{pmatrix} A_2(z) \\ B_2(z) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} A_1(z) \\ z^{-1} B_1(z) \end{pmatrix}$$

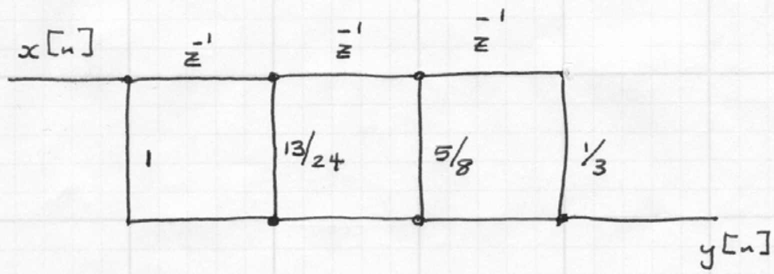
$$\begin{aligned} A_2(z) &= A_1(z) + \frac{1}{2} z^{-1} B_1(z) \\ &= 1 + \frac{1}{4} z^{-1} + \frac{1}{2} z^{-1} (\tfrac{1}{4} + z^{-1}) \\ &= 1 + \frac{3}{8} z^{-1} + \tfrac{1}{2} z^{-2} \end{aligned}$$

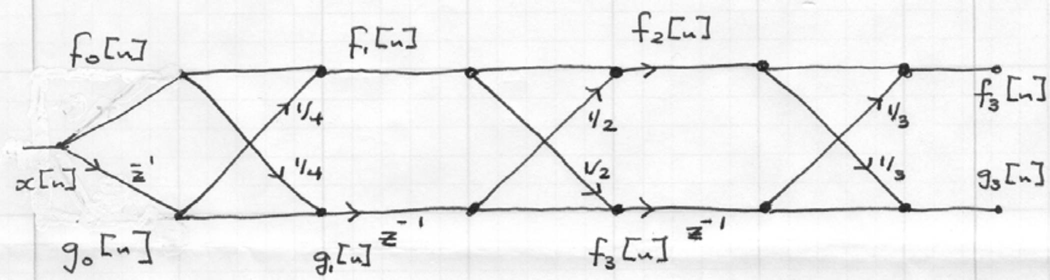
$$\begin{aligned} B_2(z) &= \tfrac{1}{2}(1 + \tfrac{1}{4} z^{-1}) + z^{-1}(\tfrac{1}{4} + z^{-1}) \\ &= \tfrac{1}{2} + \tfrac{3}{8} z^{-1} + z^{-2} \end{aligned}$$

$$A_3(z) = A_2(z) + \tfrac{1}{3} z^{-1} B_2(z)$$

$$A_3(z) = 1 + \frac{3}{8} z^{-1} + \frac{1}{2} z^{-2} + \frac{1}{3} z^{-1} \left(\frac{1}{2} + \frac{3}{8} z^{-1} + z^{-2} \right)$$

$$A_3(z) = 1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \tfrac{1}{3} z^{-3}$$





Step Down Recursion:

$$A_3(z) = 1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3}$$

$a_3(0) = 1$

$a_3(1) = 13/24$

$a_3(2) = 5/8$

$a_3(3) = k_3 = 1/3$

$j = 2$:

$a_2(0) = 1$

$B_3(z) = \frac{1}{3} + 5/8\, z^{-1} + 13/24\, z^{-2} + z^{-3}$

$$A_2(z) = \frac{1}{8/9} A_3(z) - \frac{1/3}{8/9} B_3(z)$$

$$= 1 + 3/8\, z^{-1} + 1/2\, z^{-2} \Rightarrow k_2 = 1/2$$

Order $j = 1$:

$$A_1(z) = \frac{1}{1 - 1/4} A_2(z) - \frac{1/2}{1 - 1/4} B_2(z)$$

$$= \frac{4}{3}\left(1 + 3/8\, z^{-1} + \frac{1}{2} z^{-2}\right) - \frac{2}{3}\left(\frac{1}{2} + \frac{3}{8} z^{-1} + z^{-2}\right)$$

$$= 1 + \frac{6}{24} z^{-1}$$

$$\Longrightarrow \quad K_1 = \frac{6}{24} = \frac{1}{4}$$