Example: PR Filterbank Design

Suppose the analysis filters of a two channel maximally decimated filterbank are given by the following FIR filters:

$$\begin{aligned} H_o(z) &= -6 + 2z^{-1} + 8z^{-2} - z^{-3} - 2z^{-4} = -6 + 8z^{-2} - 2z^{-4} + z^{-1}(2 - z^{-2}) \\ H_1(z) &= -3 + z^{-1} + 2z^{-2} = -3 + 2z^{-2} + z^{-1}.1 \end{aligned}$$

Extracting the polyphase components of the analysis filters we have:

$$\begin{aligned} E_{00}(z) &= -6 + 8z^{-1} - 2z^{-2}, \quad E_{01}(z) = 2 - z^{-1} \\ E_{10}(z) &= -3 + 2z^{-1}, \quad E_{11}(z) = 1. \end{aligned}$$

The corresponding type-1 polyphase matrix associated with the analysis filters if given by:

$$\mathbf{E}(z) = \begin{pmatrix} -6 + 8z^{-1} - 2z^{-2} & 2 - z^{-1} \\ -3 + 2z^{-1} & 1 \end{pmatrix}.$$

The determinant of this analysis matrix is given by:

$$\text{Det}\left(\mathbf{E}(z)\right) = -6 + 8z^{-1} - 2z^{-2} - (-3 + 2z^{-1})(2 - z^{-1}) = z^{-1}$$

Since this is a pure delay, the analysis system is a lossless system. Using the PR filterbank framework we can compute the corresponding synthesis filters that will result in a PR system:

$$\mathbf{R}(z) = cz^{-n_0} \frac{\text{Adj}(\mathbf{E}(z))}{\text{Det}(\mathbf{E}(z))}$$

If we choose $c = 1$ and $n_o = 1$ we have:

$$\mathbf{R}(z) = \text{Adj}\left(\mathbf{E}(z)\right) = \begin{pmatrix} 1 & z^{-1} - 2 \\ 3 - 2z^{-1} & -6 + 8z^{-1} - 2z^{-2} \end{pmatrix}$$

Comparing this to the standard form of the type-II polyphase matrix we can extract the corresponding synthesis filters:

$$\begin{aligned} F_0(z) &= R_{00}(z^2)z^{-1} + R_{01}(z^2) = 1z^{-1} + 3 - 2z^{-2} = 3 + z^{-1} - 2z^{-2} \\ F_1(z) &= R_{10}(z^2)z^{-1} + R_{11}(z^2) = (z^{-2} - 2)z^{-1} - 6 + 8z^{-2} - 2z^{-4} \\ &= -6 - 2z^{-1} + 8z^{-2} + z^{-3} - 2z^{-4}. \end{aligned}$$