## Lattice Structures:

Consider a sequence of FIR filters with system functions

$$H_i(z) = A_i(z) \ , \quad i = 0, 1, 2, 3, \ldots (M-1)$$

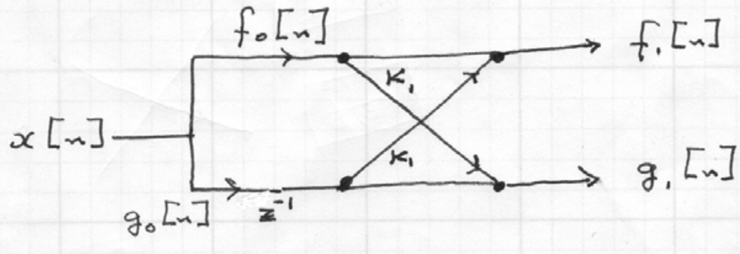
The polynomial $A_i(z)$ is defined via:

$$A_p(z) = 1 + \sum_{k=1}^{p} a_p[k]\, z^{-k}$$

Let $x[n]$ be the input to $A_p(z)$ and $y[n]$ be the output

$$y_p[n] = x[n] + \sum_{k=1}^{p} a_p[k]\, x[n-k]$$

### First-order Lattice:

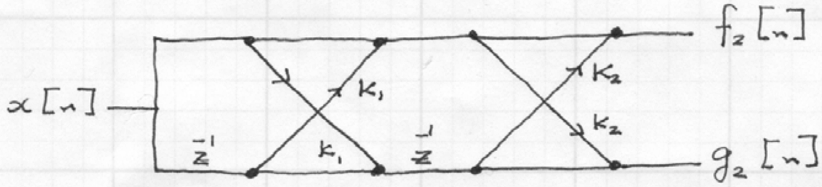



$$\left.\begin{aligned} f_1[n] &= f_0[n] + k_1 g_0[n-1] \\ g_1[n] &= g_0[n-1] + k_1 f_0[n] \end{aligned}\right\} \text{Time domain}$$

$$\frac{F_1(z)}{X(z)} = 1 + k_1 z^{-1} = 1 + a_1[1] z^{-1} = A_1(z)$$

$$\frac{G_1(z)}{X(z)} = z^{-1} + k_1 = z^{-1}(1 + k_1 z) = z^{-1} A(1/z)$$

Second Order Lattice:




$$\left.\begin{aligned} f_2[n] &= f_1[n] + k_2\, g_1[n-1] \\ g_2[n] &= g_1[n-1] + k_2\, f_1[n] \end{aligned}\right\}\ \text{Time domain}$$

Substituting $f_1[n]$ & $g_1[n]$ into equations above

$$\begin{aligned} f_2[n] = {} & f_0[n] + k_1\, g_0[n-1] \\ & + k_2\,[g_0[n-2] + k_1 f_0[n-1]] \end{aligned}$$

$$f_2[n] = x[n] + \underbrace{k_1(1+k_2)}_{a_2[1]} x[n-1] + \underbrace{k_2}_{a_2[2]} x[n-2]$$

$$\begin{aligned} a_2[1] &= k_1(1+k_2) \\ a_2[2] &= k_2 \end{aligned} \quad \left(\begin{matrix}\text{2 equations in} \\ \text{2 unknowns}\end{matrix}\right)$$

Solving the above yields

$$k_2 = a_2[2]$$

$$k_1 = \frac{a_2[1]}{1 + a_2[1]}$$

$$A_2(z) = 1 + k_1(1+k_2)z^{-1} + k_2 z^{-2} = \frac{F_2(z)}{X(z)}$$

$$g_2[n] = g_0[n-2] + k_1 f_0[n-1] + k_2 \left\{ f_0[n] + k_1 g_0[n-1] \right\}$$

$$g_2[n] = g_0[n-2] + k_1(1+k_2)\, g_0[n-1] + k_2\, g_0[n]$$

$$\frac{G_2(z)}{X(z)} = k_2 + k_1(1+k_2) z^{-1} + z^{-2}$$

$$= z^{-2}\left(1 + k_1(1+k_2) z + k_2 z^2\right)$$

$$\frac{G_2(z)}{X(z)} = z^{-2} A_2(1/z)$$

General Recursion:

$$f_j[n] = f_{j-1}[n] + k_j\, g_{j-1}[n-1]$$

$$g_j[n] = k_j\, f_{j-1}[n] + g_{j-1}[n-1]$$

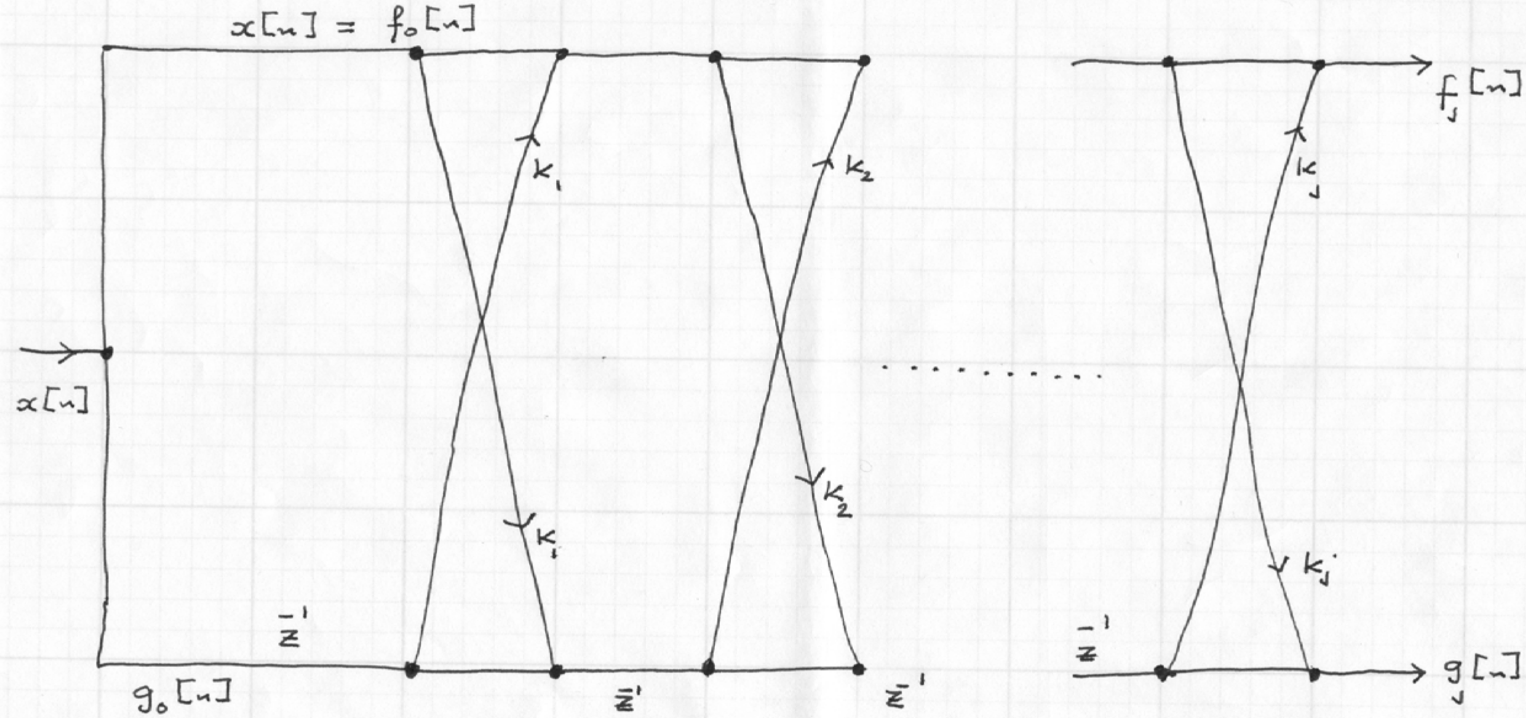
$$f_0[n] = g_0[n] = x[n]$$

Z-domain:

$$A_0(z) = \frac{F_0(z)}{X(z)} = 1$$

$$B_0(z) = \frac{G_0(z)}{X(z)} = 1$$

$$\begin{pmatrix} F_j(z) \\ G_j(z) \end{pmatrix} = \begin{pmatrix} 1 & k_j \\ k_j & 1 \end{pmatrix} \begin{pmatrix} F_{j-1}(z) \\ z^{-1} G_{j-1}(z) \end{pmatrix}$$




$$\frac{G_j(z)}{X(z)} = B_j(z) = z^{-j} A_j(z^{-1}) \quad \text{(Reverse prediction polynomial)}$$

## Recursion:

$$A_j(z) = A_{j-1}(z) + k_j z^{-1} B_{j-1}(z)$$

$$B_j(z) = k_j A_{j-1}(z) + z^{-1} B_{j-1}(z)$$

$$\begin{pmatrix} A_j(z) \\ B_j(z) \end{pmatrix} = \begin{pmatrix} 1 & k_j \\ k_j & 1 \end{pmatrix} \begin{pmatrix} A_{j-1}(z) \\ z^{-1} B_{j-1}(z) \end{pmatrix}$$

## Step down recursion:

$$A_j(z) = A_{j-1}(z) + k_j \left(B_j(z) - k_j A_{j-1}(z)\right)$$

$$A_{j-1}(z) = \frac{1}{1-k_j^2} A_j(z) - \frac{k_j}{1-k_j^2} B_j(z)$$

$$K_m = a_m[m]$$

$$a_{m-1}[0] = 1$$

$$a_{m-1}[k] = \frac{1}{1-k_m^2} a_m[k] - \frac{k_m}{1-k_m^2} b_m[k]$$

$$1 \le k \le m-1$$

- Equations are singular if $|k_m| = 1$
- Root on unit circle
- Factor out root and apply recursion to remainder