Modularity:

$$\begin{pmatrix} A_j(z) \\ B_j(z) \end{pmatrix} = \begin{pmatrix} 1 & k_j z^{-1} \\ k_j & z^{-1} \end{pmatrix} \begin{pmatrix} A_{j-1}(z) \\ B_{j-1}(z) \end{pmatrix}$$

$$\begin{pmatrix} A_1(z) \\ B_1(z) \end{pmatrix} = \begin{pmatrix} 1 & k_1 z^{-1} \\ k_1 & z^{-1} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 + k_1 z^{-1} \\ k_1 + z^{-1} \end{pmatrix}$$

$$\begin{pmatrix} A_2(z) \\ A_1(z) \end{pmatrix} = \begin{pmatrix} 1 & k_2 z^{-1} \\ k_2 & z^{-1} \end{pmatrix} \begin{pmatrix} 1 + k_1 z^{-1} \\ k_1 + z^{-1} \end{pmatrix}$$

$$A_2(z) = 1 + k_1(1+k_2) z^{-1} + k_2 z^{-2}$$

$$\begin{pmatrix} A_3(z) \\ B_3(z) \end{pmatrix} = \begin{pmatrix} 1 & k_3 z^{-1} \\ k_3 & z^{-1} \end{pmatrix} \begin{pmatrix} 1 + k_1(1+k_2) z^{-1} + k_2 z^{-2} \\ k_2 + k_1(1+k_2) z^{-1} + z^{-2} \end{pmatrix}$$

and this recursion can be continued

- To get to $A_m(z)$ from $A_{m-1}(z)$ you only need to determine k_m

Stability:

$$\prod_{i=1}^{m} r_i = k_m \quad \text{(product of roots)}$$

If system is stable $\Rightarrow |r_i| < 1, \ \forall i$

$$\Rightarrow |k_m| < 1 \quad \text{for } m = 0, \dots M-1$$

Characterestics:

(a) Lattice structure modular:

$$f_m[n] = f_{m-1}[n] + k_m g_{m-1}[n-1]$$
$$g_m[n] = k_m f_{m-1}[n] + g_{m-1}[n]$$

Addition of an extra stage does not affect earlier stages

(b) $$f_m[n] = f_{m-1}[n] + k_m g_{m-1}[n-1]$$
(Prediction error signal)

Minimizing

$E\{|f_m[n]|^2\}$ w.r.s.t k_m yields.

$$k_m = \frac{-E\{f_{m-1}[n]\, g^*_{m-1}[n-1]\}}{E\{|g_{m-1}[n-1]|^2\}}$$

$\Rightarrow$ k_m is a normalized correlation (PARCOR)

$\Rightarrow$ $|k_m| < 1$ if variances are finite

$\Rightarrow$ Monitoring stability convenient

$\Rightarrow$ $E\{|f_m[n]|^2\} < E\{|f_{m-1}[n]|^2\}$ ....

(c) Robust to finite-wordlength effects.

(d) Requires only m reflection coefficients $\{k_i\}_{i=1}^{m}$ to implement system functions $\{A_i\}_{i=1}^{m}$

(e) Convenient for slow fading communications applications (monitoring of stability easy)

and guaranteed stability for slow non stationary environments