Non Uniform FFT

$$X(z_k) = \sum_{n=0}^{N-1} x[n]\, z_k^{-n} \quad , \qquad 0 \le k \le N-1$$

$$\underbrace{\begin{pmatrix} X(z_0) \\ \vdots \\ X(z_{N-1}) \end{pmatrix}}_{\underline{X}} = \underbrace{\begin{pmatrix} 1 & z_0^{-1} & z_0^{-2} & \cdots & z_0^{-(N-1)} \\ 1 & z_1^{-1} & z_1^{-2} & \cdots & z_1^{-(N-1)} \\ \vdots & & & & \\ 1 & z_{N-1}^{-1} & z_{N-1}^{2} & \cdots & z_{N-1}^{-(N-1)} \end{pmatrix}}_{D} \underbrace{\begin{pmatrix} x[0] \\ \vdots \\ x[N-1] \end{pmatrix}}_{\underline{x}}$$

$$\underline{x} = D^{-1}\underline{X} = \frac{\mathrm{adj}(D)}{\det(D)}\underline{X}$$

$$\underline{x} = \mathrm{adj}(D)\,\underline{X} \Big/ \prod_{\substack{i,j=1 \\ i>j}}^{N} \left(z_i^{-1} - z_j^{-1}\right)$$

Implementation:

$$z_k^N\, X(z_k) = \sum_{n=0}^{N-1} x[n]\, z_k^{N-n}$$

$$z_k^N\, X(z_k) = \sum_{n=0}^{\infty} x[n]\, (z_k)^{N-n}\, u[N-n]$$

$$z_k^N X(z_k) = \sum_{n=-\infty}^{\infty} x[n]\, z_k^{r-n}\, u[r-n] \Big|_{r=N}$$

$$z_k^N X(z_k) = \sum_{\ell=-\infty}^{\infty} x[\ell]\, z_k^{r-\ell}\, u[r-\ell] \Big|_{r=N}$$

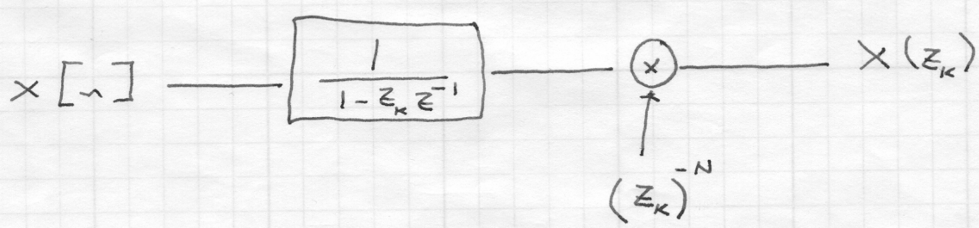
$$z_k^N X(z_k) = \sum_{\ell=-\infty}^{\infty} x[\ell]\, y_k[r-\ell] \Big|_{r=N}$$

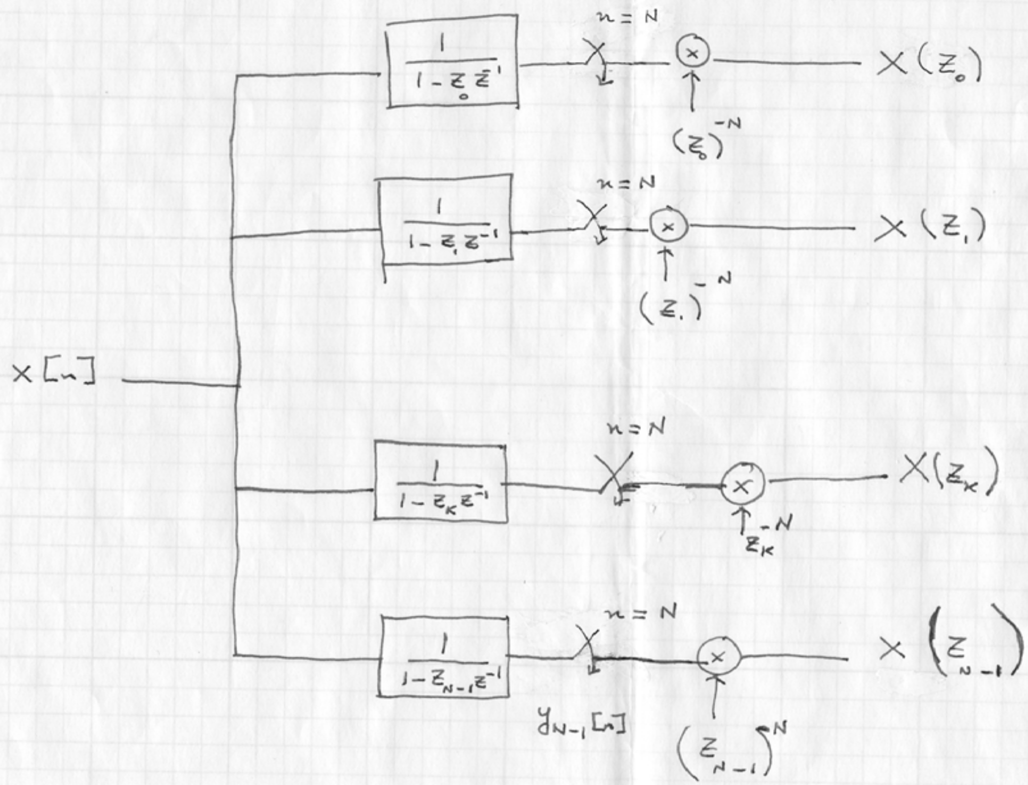
$$y_k[n] = z_k^n\, u[n]$$

$$(z_k)^N X(z_k) = x[r] * y_k[r] \Big|_{r=N}$$

$$X(z_k) = (z_k)^{-N} \left\{ x[n] * y_k[n] \right\} \Big|_{n=N}$$

$$y_k[n] = (z_k)^n\, u[n]$$





FILTER - BANK STRUCTURE

Special Case:

$$z_k = e^{j\omega_k}$$

$$H_k(z) = \frac{1}{1 - z_k z^{-1}} = \frac{1 - z_k^* z^{-1}}{1 - 2\cos(\omega_k) z^{-1} + z^{-2}}$$

$$H_k(z) = \left(\frac{1}{1 - 2\cos\omega_k z^{-1} + z^{-2}} \right) \left(1 - e^{-j\omega k} z^{-1}\right)$$

$$\frac{Q_k(z)}{X(z)} = \frac{1}{1 - 2\cos(\omega_k) z^{-1} + z^{-2}}$$

$$q_k[n] = 2\cos(\omega_k) q_k[n-1] - q_k[n-2] + x[n]$$

$$q_k[-1] = q_k[-2] = 0 \quad (IC)$$

$$\frac{X_k(z)}{Q_k(z)} = 1 - e^{-j\omega k} z^{-1}$$

$$y_k[n] = q_k[n] - e^{-j\omega_k} q_k[n-1]$$

$$X(z_k) = e^{-j\omega_k N} \left\{ q_k[N] - e^{-j\omega_k} q_k[N-1] \right\}$$