System Function Zeroes of FIR Linear Phase Systems

We saw in class the FIR linear phase systems of the types I and II have symmetric impulse responses, i.e., $h[n] = h[M-n]$ and that systems of type III and IV have antisymmetric impulse responses, i.e., $h[n] = -h[M-n]$. In this section, we will examine what these symmetry conditions mean in the Z domain.

The system function of the symmetric FIR linear phase systems is given by:

$$H(z) = \sum_{n=-\infty}^{\infty} h[M-n]z^{-n} = z^{-M}H(z^{-1}).$$

This implies that if $H(z)$ has a zero at $z = z_0$ then $H(z)$ also has a zero at $z = z_0^{-1}$. In addition if the impulse response of the system is constrained to be real then:

$$H(z) = H^*(z^*).$$

This means that the FIR system also has zeroes at $z = z_0^*$ and $z = 1/z_0^*$. This means that the zeroes of the symmetric FIR systems comes in sets of four. For the antisymmetric systems, the corresponding Z-transform relation is:

$$H(z) = \sum_{n=-\infty}^{\infty} -h[M-n]z^{-n} = -z^{-M}H(z^{-1}).$$

Specifically at $z = 1$ for the antisymmetric systems we have

$$H(1) = -(1)^{-M}H(1),$$

i.e., $H(1) = 0$ or in other words we have a zero at $\omega = 0$. In other words type III or type IV systems have a zero at $z = 1$. At $z = -1$ for the symmetric systems, we have:

$$H(-1) = (-1)^{-M}H(-1)$$

which implies that for M odd we have a zero at $z = -1$ or $\omega = \pi$. In a similar fashion we can infer that the antisymmetric systems with M even have a zero at $z = -1$ or $\omega = \pi$. In other words a type II or a type III have a zero at $z = -1$. For stable linear phase systems, we can therefore factorize the system function into three parts:

$$H_{\text{lin}}(z) = H_{\text{min}}(z)H_{\text{max}}(z)H_{\text{uc}}(z),$$

where $H_{\text{min}}(z)$ is the minimum phase part that contains those zeros and poles within the UC. $H_{\text{max}}(z)$ is the maximum phase part of $H(z)$ and

$$H_{\text{max}}(z) = z^{-M}H_{\text{min}}(z^{-1}),$$

while $H_{\text{uc}}(z)$ contains the zeroes that are located on the UC. In terms of lowpass and highpass filter design, we cannot design a lowpass system using the antisymmetric systems and we cannot design a type II highpass filter.