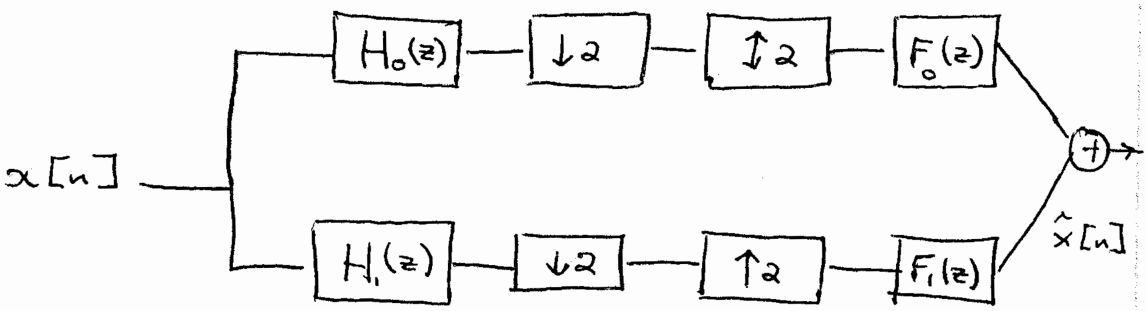
# On PR Filterbanks




PR Equations :

$$\begin{pmatrix} F_0(z) & F_1(z) \end{pmatrix} \begin{pmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{pmatrix} = \begin{pmatrix} 2c z^{-n_0} & 0 \end{pmatrix}$$

Suppose we substitute a type-I polyphase decomposition of the form :

$$H_0(z) = H_{00}(z^2) + z^{-1} H_{01}(z^2)$$
$$H_1(z) = H_{10}(z^2) + z^{-1} H_{11}(z^2)$$

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a type-II polyphase decomposition for the synthesis filters :

$$F_0(z) = F_{00}(z^2) z^{-1} + F_{01}(z^2)$$
$$F_1(z) = F_{10}(z^2) z^{-1} + F_{11}(z^2)$$

Then

$$\begin{pmatrix} F_0(z) & F_1(z) \end{pmatrix} = \begin{pmatrix} z^{-1} & 1 \end{pmatrix} \begin{pmatrix} F_{00}(z^2) & F_{10}(z^2) \\ F_{01}(z^2) & F_{11}(z^2) \end{pmatrix}$$

$$\begin{pmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{pmatrix}$$

$$= \begin{pmatrix} H_{00}(z^2) & H_{01}(z^2) \\ H_{10}(z^2) & H_{11}(z^2) \end{pmatrix} \begin{pmatrix} 1 & 1 \\ z^{-1} & -z^{-1} \end{pmatrix}$$

$$[F_0(z) \;\; F_1(z)] \begin{pmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{pmatrix}$$

$$= (z^{-1} \;\; 1) \begin{pmatrix} F_{00}(z^2) & F_{10}(z^2) \\ F_{01}(z^2) & F_{11}(z^2) \end{pmatrix} \begin{pmatrix} H_{00}(z^2) & H_{01}(z^2) \\ H_{10}(z^2) & H_{11}(z^2) \end{pmatrix} \begin{pmatrix} 1 & 1 \\ z^{-1} & -z^{-1} \end{pmatrix}$$

Suppose we design the filters $\ni$

$$\begin{pmatrix} F_{00}(z^2) & F_{10}(z^2) \\ F_{01}(z^2) & F_{11}(z^2) \end{pmatrix} \begin{pmatrix} H_{00}(z^2) & H_{01}(z^2) \\ H_{10}(z^2) & H_{11}(z^2) \end{pmatrix} = I$$

then

$$[F_0(z) \;\; F_1(z)] \begin{pmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{pmatrix} = \begin{pmatrix} 2z^{-1} & 0 \end{pmatrix}$$

$\Rightarrow$ A sufficient condition for PR criteria is :

$$\mathbb{F}(z^2) \; \mathbb{H}(z^2) = I_{2\times 2} \quad \text{or}$$

$$\mathbb{F}(z) \; \mathbb{H}(z) = I_{2\times 2} ,$$

where $\mathbb{F}(z) \triangleq \begin{pmatrix} F_{00}(z) & F_{10}(z) \\ F_{01}(z) & F_{11}(z) \end{pmatrix}$ is

the type-II polyphase synthesis matrix

and $\mathbb{H}(z) \triangleq \begin{pmatrix} H_{00}(z) & H_{01}(z) \\ H_{10}(z) & H_{11}(z) \end{pmatrix}$ is

the type-I polyphase analysis matrix

A generalization of the PR condition is to allow:

$$\mathbb{F}(z)\,\mathbb{H}(z) = c\,z^{-m_0}\, I_{2\times 2}$$

or

$$\mathbb{F}(z)\,\mathbb{H}(z) = c\,z^{-m_0} \begin{pmatrix} 0 & 1 \\ z^{-1} & 0 \end{pmatrix}$$

It can be easily verified that either of these will result in a PR filterbank also