ECE-539, SPRING-09

Digital Signal Processing

Example: Parallel form

Consider the causal and stable system function:

$$H(z) = \frac{1}{(1-p_1 z^{-1})(1-p_2 z^{-1})}, \quad |z| > \max\{|p_1|, |p_2|\}$$

Expanding $H(z)$ using a partial fraction decomposition:

$$H(z) = \frac{k_1}{1-p_1 z^{-1}} + \frac{k_2}{1-p_2 z^{-1}}$$

Comparing coefficients of z^0, z^{-1}:

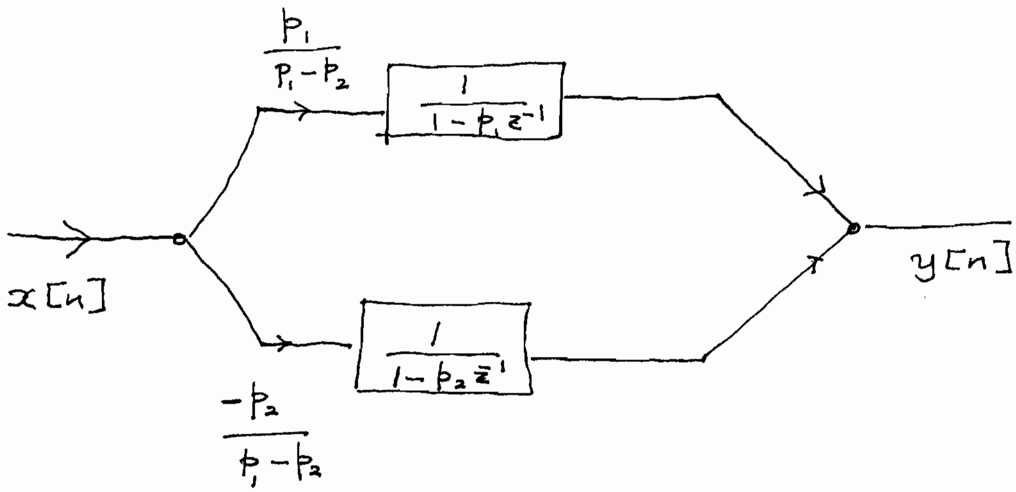
$$k_1 + k_2 = 1 \qquad (1)$$

$$-k_1 p_2 - k_2 p_1 = 0 \qquad (2)$$

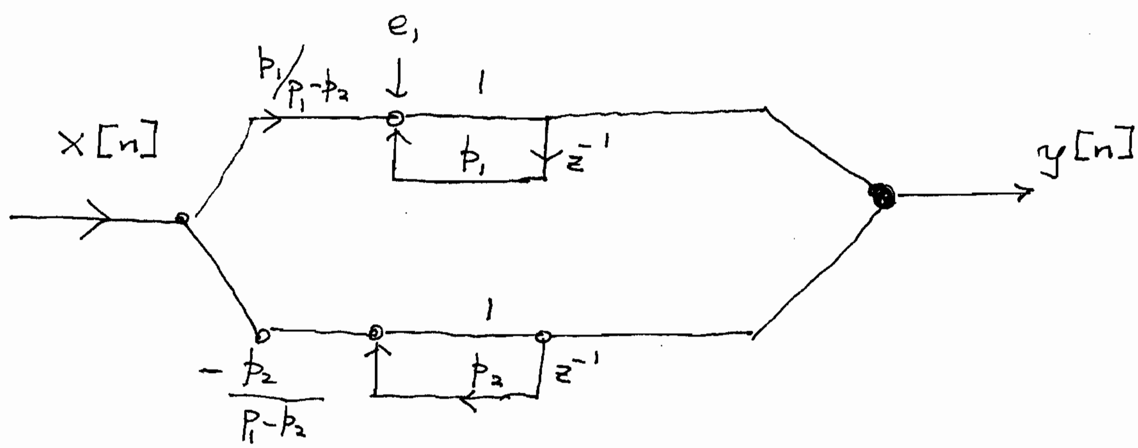
Solving these simultaneously:

$$k_2 = -k_1 \frac{p_2}{p_1} \implies k_1 \left(1 - \frac{p_2}{p_1}\right) = 1$$

$$k_1 = \frac{p_1}{p_1 - p_2}, \quad k_2 = \frac{-p_2}{p_1 - p_2}$$



Finite Precision model:



Output Quantization Noise:

$$\sigma_f^2 = \frac{2\sigma_e^2}{1-p_1^2} + \frac{2\sigma_e^2}{1-p_2^2}$$

Additionally

$$\frac{\hat{k}_1}{1-\hat{p}_1 z^{-1}} + \frac{\hat{k}_2}{1-p_2 z^{-1}} = \frac{\hat{k}_1 + \hat{k}_2 - z^{-1}(\hat{k}_1 p_2 + \hat{k}_2 p_1)}{(1-\hat{p}_1 z^{-1})(1-\hat{p}_2 z^{-1})}$$

A zero is now present at $z = \dfrac{\hat{k}_1 p_2 + \hat{k}_2 p_1}{\hat{k}_1 + \hat{k}_2}$

Salient Features:

(a) The parallel form implements sections that are independent of each other in contrast to the direct form.

(b) The parallel form may not be desirable if strict control over zero locations is needed.

(c) Scaling in the front-end before SOP quantization will reduce output quantization noise power, while decreasing input SNR ($k_1 + k_2 = 1$)

(d) Placement of scalar multipliers would depend on magnitude of the multiplier. Large numbers at the end would magnify noise, while large numbers in the front would cause input overflow.