ECE-539, Spring 2009
Digital Signal Processing

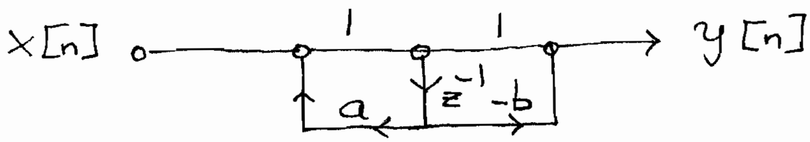
Example : Pole-zero combination

Consider the causal & stable system with system function

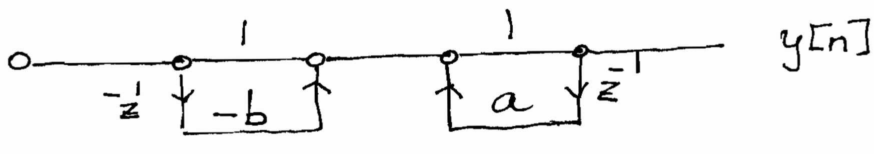
$$H(z) = \frac{1-bz^{-1}}{1-az^{-1}}, \quad |z| > |a|$$

The direct form implementation:

Type - II



Type - I



For type - II :

$$\sigma_f^2 = \sigma_e \sum_{n=-\infty}^{\infty} h^2[n] + \sigma_e^2$$

For type I :

$$\sigma_f^2 = \frac{2\sigma_e^2}{1-a^2}$$

$$h[n] = a^n u[n] - b a^{n-1} u[n-1]$$

$$h[0] = 1$$

$$h[1] = a - b$$

$$h[2] = a^2 - ba = a(a-b)$$

$$h[3] = a^3 - ba^2 = a^2(a-b)$$

$$\sum_{n=-\infty}^{\infty} h^2[n] = 1 + (a-b)^2 + a^2(a-b)^2 + a^4(a-b)^2 + \cdots\cdots$$

$$= 1 + (a-b)^2 \cdot \frac{1}{1-a^2}$$

$$= 1 + \frac{a^2+b^2-2ab}{1-a^2}$$

$$\sigma_f^2 = \sigma_e^2\left(1 + \frac{a^2+b^2-2ab}{1-a^2} + 1\right)$$

$$= \sigma_e^2\left(2 + \frac{a^2+b^2-2ab}{1-a^2}\right)$$

$$= \sigma_e^2\left(\frac{2-a^2+b^2-2ab}{1-a^2}\right)$$

More importanty :

$$\lim_{\substack{a\to 1\\ b\to 1}} \sigma_f^2 = 2\sigma_e^2 \neq \infty$$

Salient features:

(a) Normaly quantization of pole close to UC will result in noise amplification

(b) Combining a pole close to UC with a zero close to UC reduces the detrimental effects of the pole

(c) Only applies to type-II structures.