PS #1 , Spring 2015
Digital Signal Processing, ECE-539
Instructor: Balu Santhanam
Date Assigned: 01/15/2015
Date Due: 01/27/2015

Problem # 1.0

Consider the ideal, zero-phase, discrete-time, lowpass filter with a passband gain of $G = L$ and a cut-offfrequency $\omega_c = \frac{\pi}{L}$.

1. Calculate the impulse response of this lowpass filtter $h_{lp}[n]$.
2. Suppose we define a sequence of functions $\phi_k[n] = h_{lp}[n - kL]$. Show that this sequence is a pair-wise orthogonal sequence.
3. Is this sequence a basis for some space of discrete-time signals? If so are there other basis possible for the same space.

Problem # 2.0

Suppose we have a finite-energy, time-limited signal $x_c(t)$ that is supported on the interval $t \in [-t_o, t_o]$. We wish to sample this signal in frequency by modeling the sampling process as:

$$X_s(j\Omega) = X_c(j\Omega)P(j\Omega),$$

where $P(j\Omega)$ is the periodic impulse train signal in frequency with a teeth-spacing of Ω_s. Derive the sampling theorem results and the reconstruction result for this frequency-domain sampling theorem problem.

Problem # 3.0

In class, when we derived the Nyquist sampling theorem, we assumed an ideal impulse train to model uniform sampling. In reality, however, we need to approximate the impulse train with a rectangular pulse train with duty cycle $D = \tau/T_s$. Derive the sampling theorem results for this realistic pulse train.

Problem # 4.0

Consider the upsampling and downsampling systems given by:

$$y_1[n] = L_1(x[n]) = x[n/L], \quad y_2[n] = L_2(x[n]) = x[Mn].$$

For these operations: (a) show that underlying systems are linear time-varying systems and (b) determine the time-varying impulse response $h[m, n]$ associated with them.