Radix-- M FFT Algorithm

$$N = M^{\gamma} \quad (\text{Size of FFT})$$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

Specifically $M = 3$

$$X[k] = \sum_{n=0}^{N/3-1} x[n] W_N^{nk} + \sum_{n=N/3}^{2N/3-1} x[n] W_N^{nk} + \sum_{n=2N/3}^{N-1} x[n] W_N^{nk}$$

$$X[3k] = \sum_{n=0}^{N/3-1} x[n] W_{N/3}^{nk} + \sum_{n=0}^{N/3-1} x[n+\tfrac{N}{3}] W_{N/3}^{nk} + \sum_{n=0}^{N/3-1} x[n+\tfrac{2N}{3}] W_{N/3}^{nk}$$

$$X[3k] = \sum_{n=0}^{N/3-1} \left(x[n] + x[n+N/3] + x[n+\tfrac{2N}{3}] \right) W_{N/3}^{nk}$$

$$X[3k+1] = \sum_{n=0}^{N/3-1} x[n] W_N^{(3k+1)n} + \sum_{n=0}^{N/3-1} x[n+\tfrac{N}{3}] W_N^{(3k+1)(n+\frac{N}{3})} + \sum_{n=0}^{N/3-1} x[n+\tfrac{2N}{3}] W_N^{(3k+1)(n+2N/3)}$$

$$X[3k+1] = \sum_{n=0}^{N/3-1} \left\{ x[n] W_N^n + x[n+N/3] W_N^n W_3^1 + x[n+\tfrac{2N}{3}] W_N^n W_3^2 \right\} W_{N/3}^{nk}$$

$$X[3k+1] = \sum_{n=0}^{N/3-1} \left(x[n] + x[n+\tfrac{N}{3}] W_3^1 + x[n+\tfrac{2N}{3}] W_3^2 \right) W_N^n W_{N/3}^{nk}$$

$$X[3k+2] = \sum_{n=0}^{N/3-1} \left\{ x[n] + x[n+\tfrac{N}{3}] W_3^2 + x[n+\tfrac{2N}{3}] W_3^1 \right\} W_N^{2n} W_{N/3}^{nk}$$

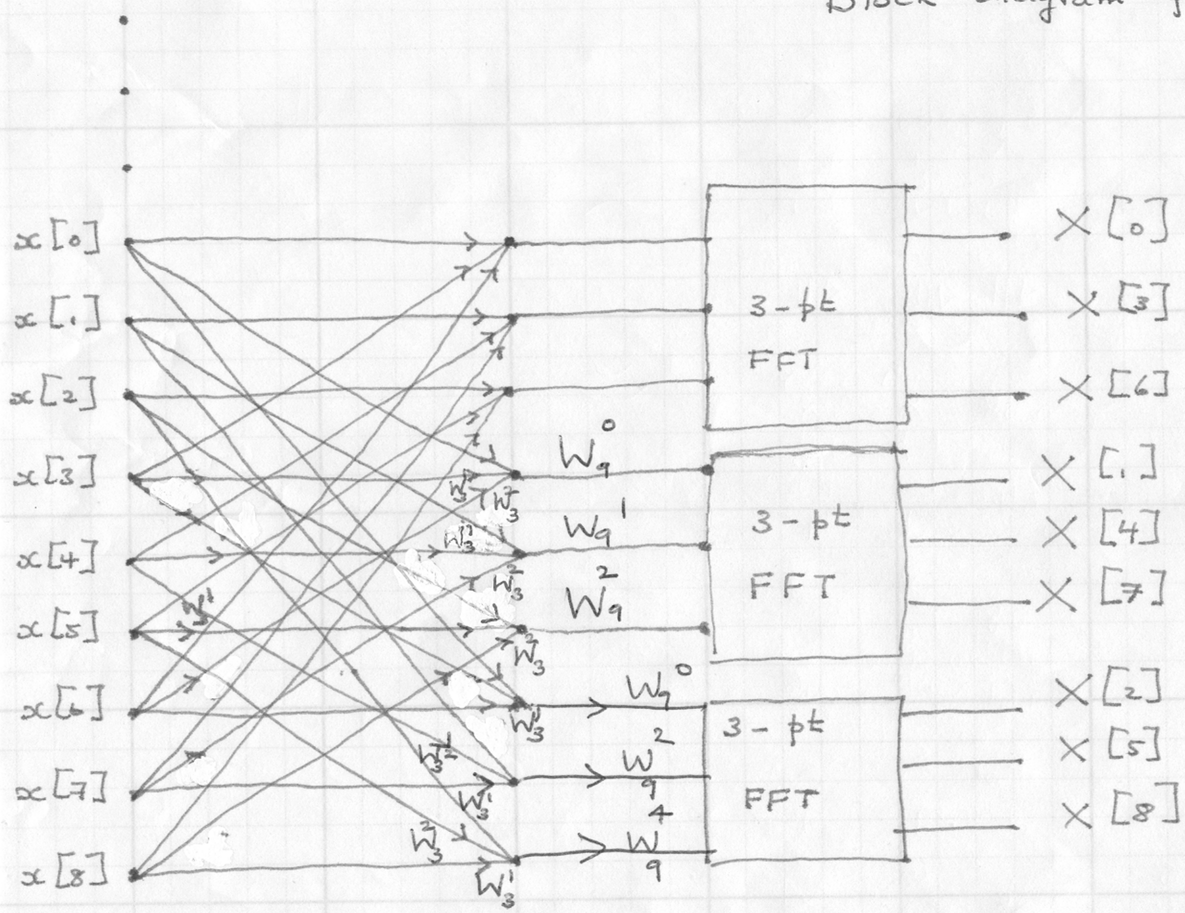
$$p[n] \triangleq x[n] + x[n+N/3] + x[n+2N/3]$$

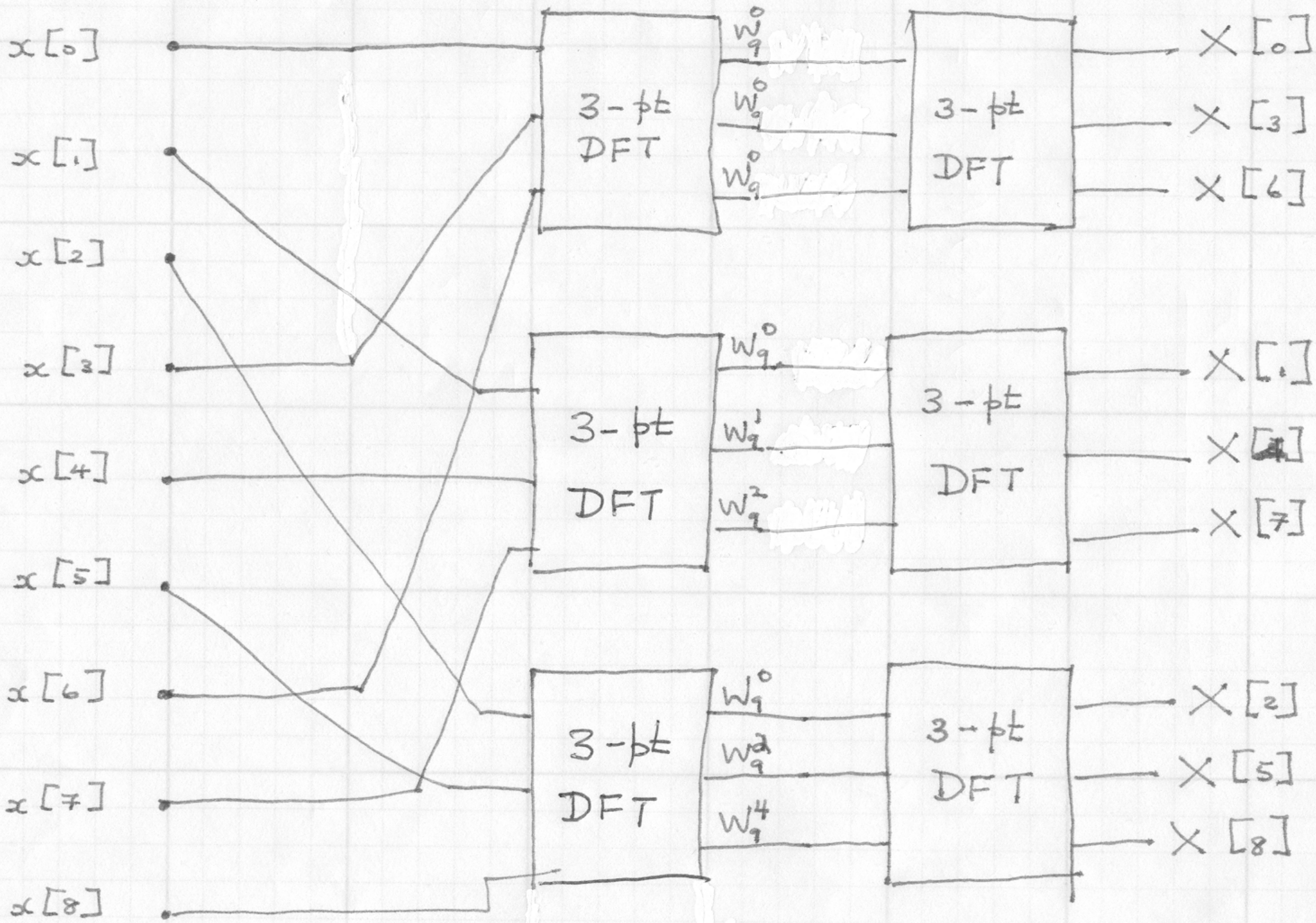
$$q[n] \triangleq \left\{ x[n] + x[n+\tfrac{N}{3}] W_3^1 + x[n+\tfrac{2N}{3}] W_3^2 \right\} W_N^n$$

$$r[n] \triangleq \left\{ x[n] + x[n+\tfrac{N}{3}] W_3^2 + x[n+\tfrac{2N}{3}] W_3^1 \right\} W_N^{2n}$$

$$\left.\begin{aligned} X[3k] &= P[k] \\ X[3k+1] &= Q[k] \\ X[3k+2] &= R[k] \end{aligned}\right\} \frac{N}{3} \text{ pt transforms}$$

Block diagram for N = 9 FFT





x[0]
x[1]
x[2]
x[3]
x[4]
x[5]
x[6]
x[7]
x[8]
3-pt DFT
3-pt DFT
3-pt DFT
W_9^0
W_9^0
W_9^0
W_9^0
W_9^1
W_9^2
W_9^0
W_9^2
W_9^4
3-pt DFT
3-pt DFT
3-pt DFT
X[0]
X[3]
X[6]
X[1]
X[4]
X[7]
X[2]
X[5]
X[8]

Computational Complexity

$$O(N) = \left(\frac{\text{\# of multip}}{\text{3-pt DFT}}\right)\left(\frac{\text{\# of DFT's}}{\text{Stage}}\right)\left(\begin{array}{c}\text{\# of stages}\\ \text{with DFT's}\end{array}\right)$$

$$+ \frac{\text{\# of twiddle fac}}{\text{stage}}\left(\begin{array}{c}\text{\# of stages}\\ \text{with twiddle}\\ \text{factors}\end{array}\right)$$

$$O(N) = (4)(\gamma)(3)^{\gamma-1} + N(\gamma-1)$$

$$= 4 \log_3 N \ \frac{N}{3} + N(\log_3 N - 1)$$

$$= \frac{7N}{3} \log_3 N - N$$

For $N = 9 = 3^2$

$$O(9) = (21)\log_3 9 - 9$$

$$= (21)(2) - 9 = 33$$

For direct computation: $O_{(2)}(N) = 81$