ECE-539, Spring 2009
Digital Signal Processing

Example: Schur-Cohn Stability

Consider the causal LTI system:

$$A_3(z) = 1 + \frac{13}{24} z^{-1} + \frac{5}{8} z^{-2} + \frac{1}{3} z^{-3}$$
$$B_3(z) = \frac{1}{3} + \frac{5}{8} z^{-1} + \frac{13}{24} z^{-2} + z^{-3}$$

$$\begin{bmatrix} A_2(z) \\ B_2(z) \end{bmatrix} = \begin{pmatrix} 1 & k_3 z^{-1} \\ k_3 & z^{-1} \end{pmatrix}^{-1} \begin{bmatrix} A_3(z) \\ B_3(z) \end{bmatrix}$$

$$a_3[3] = k_3 = \frac{1}{3}$$

$$A_2(z) = \frac{-k_3}{1-k_3^2} B_3(z) + \frac{1}{1-k_3^2} A_3(z)$$

$$A_2(z) = \frac{-\frac{1}{3}}{1-\frac{1}{9}} \begin{pmatrix} \frac{1}{3} \\ \frac{5}{8} \\ 13/24 \\ 1 \end{pmatrix} + \frac{1}{1-\frac{1}{9}} \begin{pmatrix} 1 \\ 13/24 \\ 5/8 \\ 1/3 \end{pmatrix}$$

$$A_2(z) = -\frac{3}{8} \begin{pmatrix} \frac{1}{3} \\ 5/8 \\ 13/24 \\ 1 \end{pmatrix} + \frac{9}{8} \begin{pmatrix} 1 \\ 13/24 \\ 5/8 \\ 1/3 \end{pmatrix}$$

$$A_2(z) = \begin{pmatrix} 1 \\ 3/8 \\ \frac{1}{2} \\ 0 \end{pmatrix} = 1 + \frac{3}{8} z^{-1} + \frac{1}{2} z^{-2}$$

$$a_2[2] = K_2 = \frac{1}{2}$$

$$B_2(z) = \frac{1}{2} + \frac{3}{8} z^{-1} + z^{-3}$$

$$A_1(z) = \frac{-\frac{1}{2}}{1-\frac{1}{4}} \begin{pmatrix} \frac{1}{2} \\ \frac{3}{8} \\ 1 \end{pmatrix} + \frac{1}{1-\frac{1}{4}} \begin{pmatrix} 1 \\ \frac{3}{8} \\ \frac{1}{2} \end{pmatrix}$$

$$A_1(z) = -\frac{2}{3} \begin{pmatrix} \frac{1}{2} \\ 3/8 \\ 1 \end{pmatrix} + \frac{4}{3} \begin{pmatrix} 1 \\ 3/8 \\ \frac{1}{2} \end{pmatrix}$$

$$A_1(z) = \begin{pmatrix} 1 \\ \frac{1}{4} \\ 0 \end{pmatrix} = 1 + \frac{1}{4} z^{-1}$$

$$a_1[1] = K_1 = \frac{1}{4}$$

The reflection coefficients associated with $A_3(z)$ are:

$$\left.\begin{aligned} K_1 &= 1/4 \\ k_2 &= 1/2 \\ k_3 &= 1/3 \end{aligned}\right\} \; |k_i| < 1, \; i = 1, 2, 3$$

$\Longrightarrow$ $A_3(z)$ is a minimum-phase polynomial.