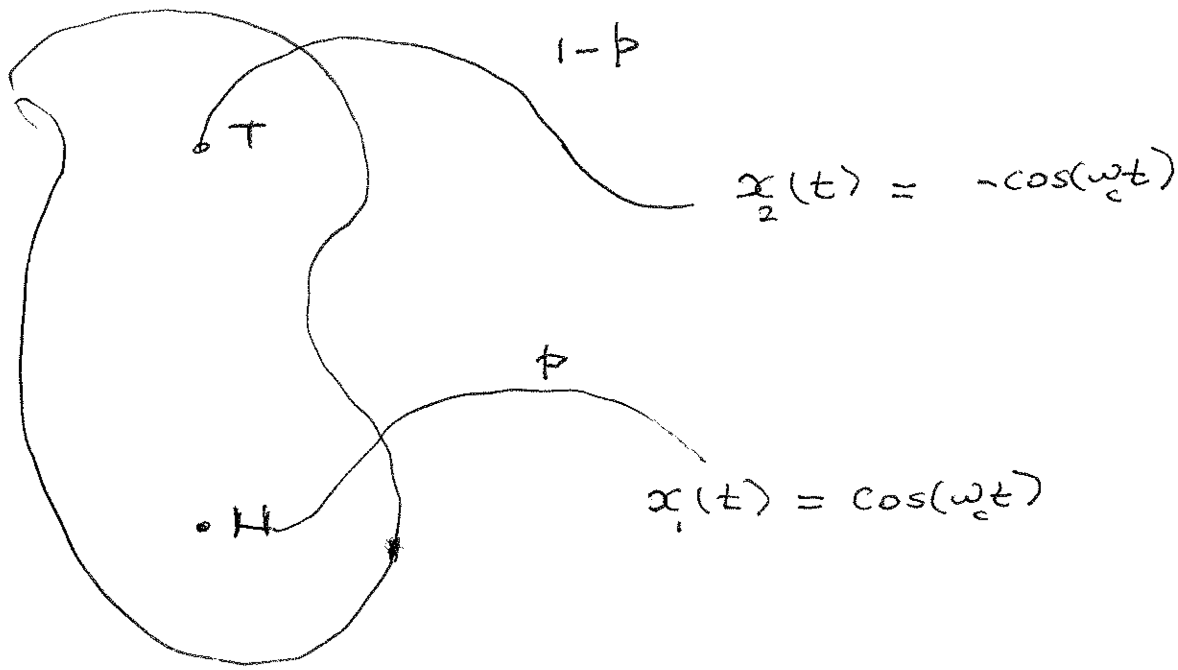
ECE-541, FALL 2018

Probability Theory & Stoc. Proc.

Example: Cyclostationarity



First-order Statistics:

(i) $\mu_X(t) = E\{X_t(\omega)\}$

$$= E\{A(\omega)\cos(\omega_c t)\},$$

$A(\omega)$ is Bernoulli $\in \{\underset{p}{1}, \underset{1-p}{-1}\}$

$$\mu_x(t) = p\cos(\omega_c t) + q(-\cos(\omega_c t))$$

$$= (p-q)\cos(\omega_c t)$$

$$E\{X_t^2(\omega)\} = p\cos^2(\omega_c t) + q(-\cos(\omega_c t))^2$$

$$= (p+q)\cos^2(\omega_c t)$$

$$\sigma_x^2(t) = (p+q)\cos^2(\omega_c t) - (p-q)^2\cos^2(\omega_c t)$$

$$= [(p+q) - (p-q)^2]\cos^2(\omega_c t)$$

$$= (1-(p-q)^2)\cos^2(\omega_c t)$$

The first-order statistics are periodic and the process is $n=1$-order cyclostationary with

$$T_0^{(1)} = \max\left\{\frac{2\pi}{\omega_c}, \frac{2\pi}{2\omega_c}\right\}$$

$$= \frac{2\pi}{\omega_c}$$

Second-order Statistics

$$R_{xx}(t_1, t_2) = E_A\{A(\omega)\cos(\omega_c t_1)A(\omega)\cos(\omega_c t_2)\}$$

$$= E\{A^2(\omega)\}\cos(\omega_c t_1)\cos(\omega_c t_2)$$

$$E\{A^2(\omega)\} = p(+1)^2 + q(-1)^2 = 1$$

$$R_{xx}(t_1, t_2) = \cos(\omega_c t_1)\cos(\omega_c t_2)$$

$$C_{xx}(t_1, t_2) = \cos(\omega_c t_1)\cos(\omega_c t_2) - (p-q)^2 \cos(\omega_c t_1)\cos(\omega_c t_2)$$

$$= (1-(p-q)^2)\cos(\omega_c t_1)\cos(\omega_c t_2)$$

$$\rho_{xx}(t_1, t_2) = \frac{\cancel{(1-(p-q)^2)}\cos(\omega_c t_1)\cos(\omega_c t_2)}{\cancel{(1-(p-q)^2)}\,|\cos(\omega_c t_1)\cos(\omega_c t_2)|}$$

$$= \frac{\cos(\omega_c t_1)\cos(\omega_c t_2)}{|\cos(\omega_c t_1)\cos(\omega_c t_2)|}$$

$$C_{xx}(t_1, t_2) = C_{xx}\left(t_1 + \frac{2\pi}{\omega_c}, t_2 + \frac{2\pi}{\omega_c}\right)$$

$$R_{xx}(t, t-\tau) = \frac{1}{2}\left\{\cos(2\omega_c t - \omega_c \tau) + \cos(\omega_c \tau)\right\}$$

$$R_{xx}(t, t-\tau) = \frac{1}{2}\cos(\omega_c \tau) + \frac{1}{2}\exp(j2\omega_c t - j\omega_c \tau) + \frac{1}{2}\exp(-j2\omega_c t + j\omega_c \tau)$$

$$T_0^{(2)} = \frac{2\pi}{2\omega_c}$$

$$R_{xx}^{(n)}(\tau) = \begin{cases} \frac{1}{2}\cos(\omega_c \tau), & n = 0 \\ \frac{1}{2} e^{-j\omega_c \tau}, & n = 1 \\ \frac{1}{2} e^{j\omega_c \tau}, & n = -1 \end{cases}$$

$$\begin{aligned} R_{xx}^{(1)}(\tau) + R_{xx}^{(-1)}(\tau) &= \cos(\omega_c \tau) \\ &= \left[\frac{1}{2}\cos(\omega_c \tau)\right] \cdot 2 \\ &= 2 R_{xx}^{(0)}(\tau) \end{aligned}$$

$$P_{xx}^{(n)}(j\Omega) = \begin{cases} \frac{\pi}{2}\delta(\Omega - \omega_c) + \frac{\pi}{2}\delta(\Omega + \omega_c) & n = 0 \\ \pi\,\delta(\Omega + \omega_c), & n = 1 \\ \pi\,\delta(\Omega - \omega_c), & n = -1 \end{cases}$$

$$P_{xx}^{(1)}(j\Omega) + P_{xx}^{(-1)}(j\Omega) = 2 P_{xx}^{(0)}(j\Omega)$$