

## Filtering of Random Signals

Consider a discrete-time LTI system with a system function of the form:

$$H(z) = \frac{b}{1 - az^{-1}}, \quad |z| > |a|, \quad |a| < 1, \quad a \in \mathbf{R}^1.$$

A zero-mean, WSS, variance  $\sigma^2$  white Gaussian random signal  $x[n]$  is the input to the system  $H(z)$  producing an output signal  $y[n]$ . It can be shown that the ACF of the output process is:

$$R_{yy}[k] = \frac{\sigma^2 b^2}{1 - a^2} a^{|k|}, \quad k \in \mathbf{I}.$$

The PSD of the output  $y[n]$  in this case can be shown to be:

$$P_{yy}(z) = \frac{\sigma^2 b^2}{(1 - az^{-1})(1 - az)}, \quad a < |z| < \frac{1}{a}.$$

On the unit circle, this expression for the PSD reduces to:

$$P_{yy}(e^{j\omega}) = \frac{\sigma^2 b^2}{1 - 2a \cos(\omega) + a^2}, \quad \omega \in [-\pi, \pi].$$

The average power of the output random signal is therefore:

$$P_{\text{ave}}^y = \frac{\sigma^2 b^2}{1 - a^2}.$$

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%*****
% MATLAB MACRO FOR FILTERING OF RANDOM SIGNALS
% Author: Balu Santhanam
% Date : 10/12/2001
% EECE-541, Fall 2001
%*****
format long
% Input signal is a zero-mean, WSS, Gaussian random signal
% with unit variance
N = 2000; x = awn(zeros(1,N), [0,1], 'AWGN');
% Filter signal through a LTI system
b = 1; a = [1,-0.5]; y = filter(b,a,x);
% Estimate output ACF and PSD
R_yy = xcorr(y, 'coeff');
[F,P_yy] = psd(y,512,1,128,1);
P_act = abs(fft(b,512) ./ fft(a,512)).^2;
%*****

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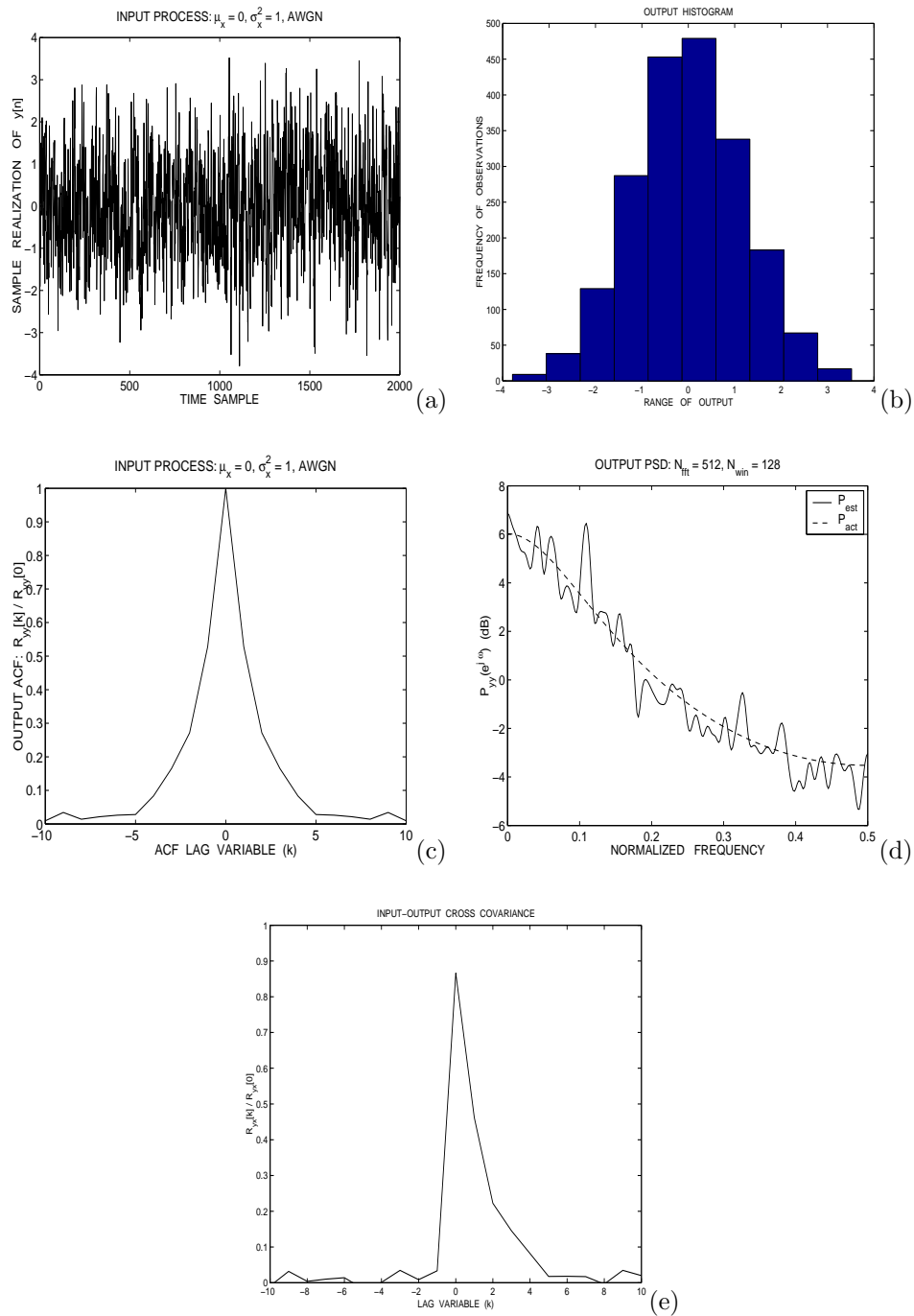


Figure 1: Filtering of white noise: (a) sample realization of  $y[n]$ , (b) histogram of the output process  $y[n]$ , (c) output ACF, (d) output PSD, (e) output-input cross-covariance.