Random Processes: Notation and Definitions

Consider the probability space, $(\Omega, \mathcal{F}, P)$, where we have seen that random variables can be defined as measurable transformations on this space, i.e., $X : \Omega \to \mathbf{R}^1$. We saw that the transformation is measurable only if the inverse image belongs to the Borel field of events. Instead of mapping the events in the field $\mathcal{F}$ to the real-line if we map these events to a set of functions then the corresponding transformation will be refered to as a *random process*. A random process $X(t)$ can be viewed or defined in three ways:

- A *random process* $X(t)$ is a time indexed collection of random variables defined on $(\Omega, \mathcal{F}, P)$ and is denoted $X_t(\omega)$. For every instant $t_o \in \mathbf{T} \subseteq \mathbf{R}^1$, we obtain a R.V denoted $X_{t_o}(\omega)$. With this interpretation the quantity ω is the variable and the quantity t is the parameter.

- The alternative definition for a random process is that it is an ensemble or collection of waveforms indexed by the parameter ω, i.e., $X_\omega(t)$. For every $\omega_o \in \Omega$, we obtain a time waveform $X_{\omega_o}(t)$. The time function obtained will be referred to as a *sample function* or *ensemble waveform*. In this case the quantity t is the variable and ω becomes the parameter

- Of course the random process $X(t)$ can also be viewed as a measurable two-variable function $X(t, \omega)$ defined on the Cartesian product space $T \times \Omega$, where the inverse image mapping the *member functions* back to the sample space Ω, lies in the Borel field of events, i.e.,

$$X^{-1}\{x_\omega(t), \omega \in \Omega\} \in \mathcal{F}.$$

Other terms used to refer to random processes are "random signals" or "stochastic processes". The set of values that a random process can take is referred to as the *state space* of the random process. If the state space of the random process is a countable subset of the real line then the process is called a *discrete-amplitude*. If the state-space of the random process is a dense subset of the real line then we call the process a *continuous-amplitude* random process. If the indexing set $\mathbf{T}$ is a countable subset of the real line then we are dealing with a *discrete-time* random process. If the indexing set $\mathbf{T}$ is a dense subset of the real-line then we call the process a *continuous-time* random process. For practical purposes, however, we will only consider a finite number of sampling points. As we will see later, the sampling theorem can be used to represent a continuous–time random process in terms of its sampled versions later in the course.